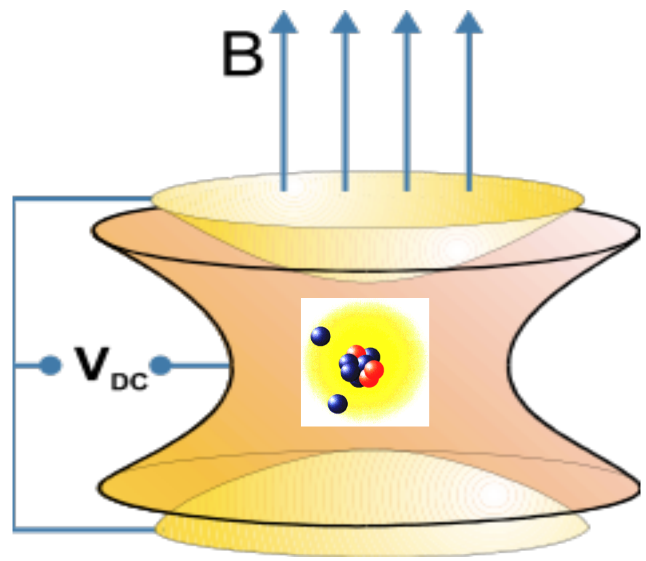
CANADA'S NATIONAL LABORATORY FOR PARTICLE AND NUCLEAR PHYSICS
Owned and operated as a joint venture by a consortium of Canadian universities via a contribution through the National Research Council Canada

First direct mass measurement of the two and four neutron halos ^{6}He and ^{8}He using the TITAN Penning trap mass spectrometer



TITAN
ISAC-TRIUMF

Maxime Brodeur
UBC Ph.D. candidate

LABORATOIRE NATIONAL CANADIEN POUR LA RECHERCHE EN PHYSIQUE NUCLÉAIRE ET EN PHYSIQUE DES PARTICULES
Propriété d'un consortium d'universités canadiennes, géré en co-entreprise à partir d'une contribution administrée par le Conseil national de recherches Canada

Outline

1) Halo nuclei characteristics

2) Motivations for ^{6,8}He mass measurement

3) Experimental method

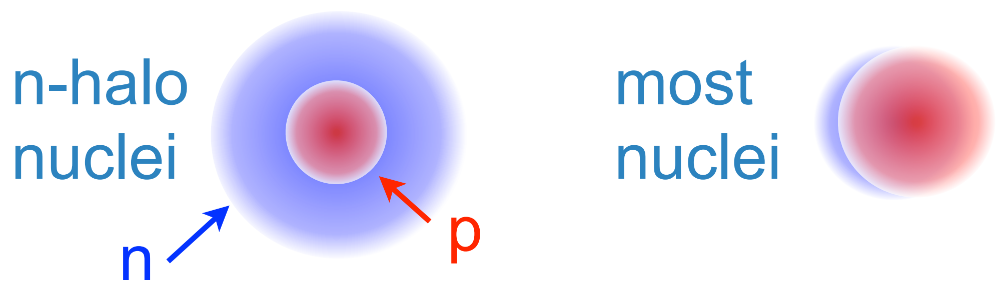
4) Results and discussion

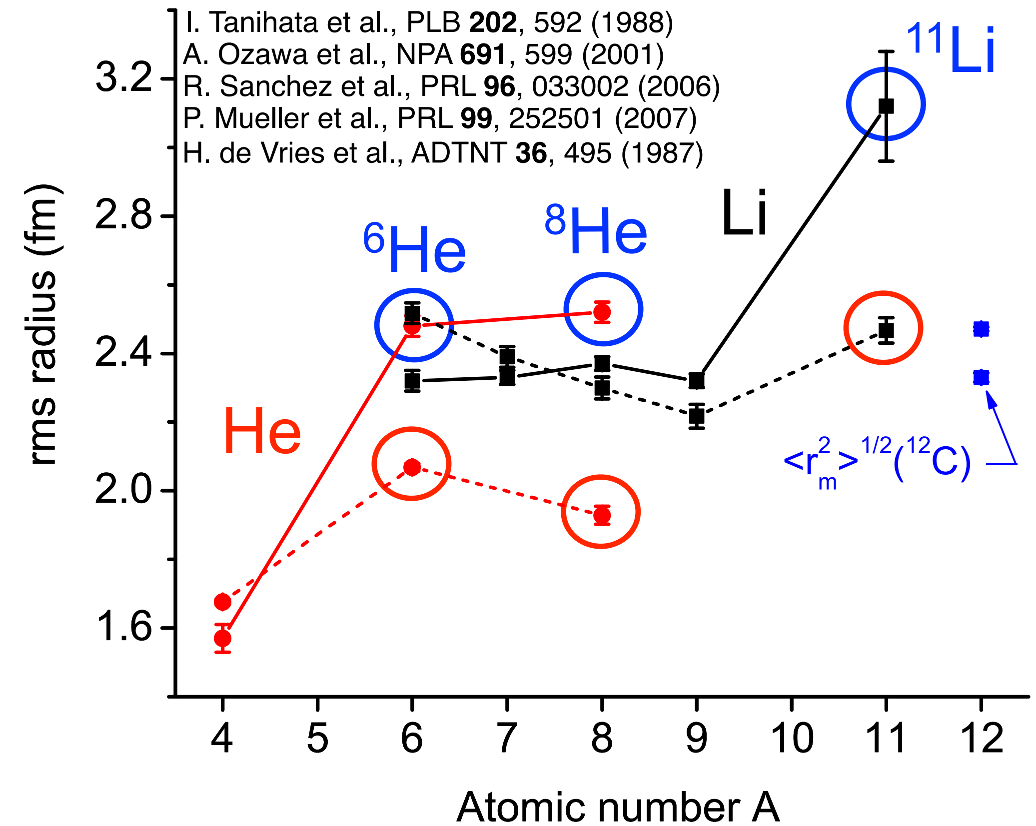
5) Summary and outlook

Halo nuclei properties

→ Halos are untypical nuclei characterized by:

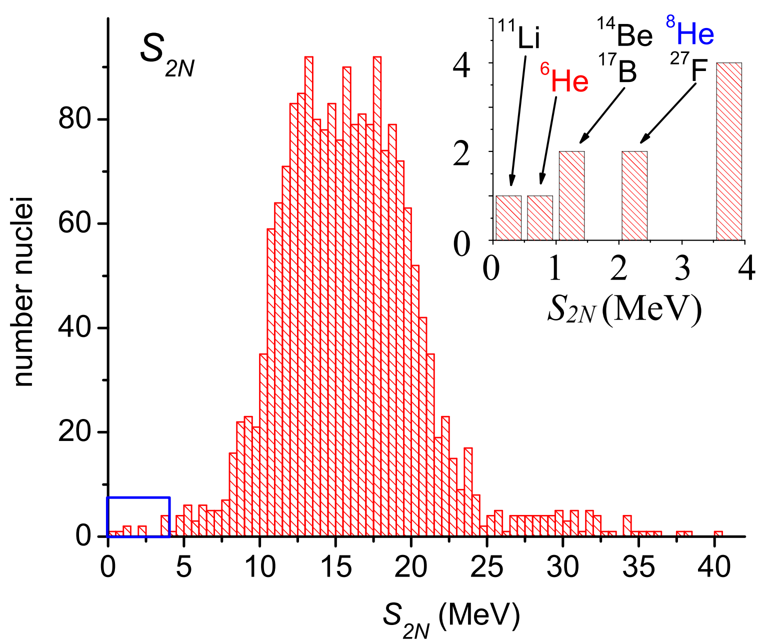
- Large matter distribution (larger than the usual $A^{1/3}$ dependence)
- Large difference in charge and matter radii

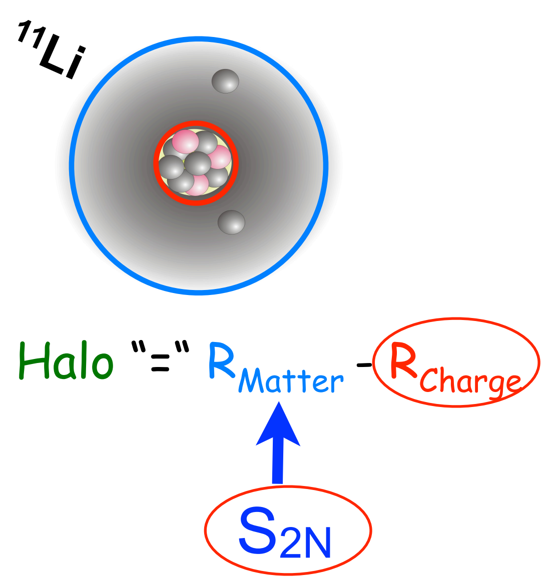


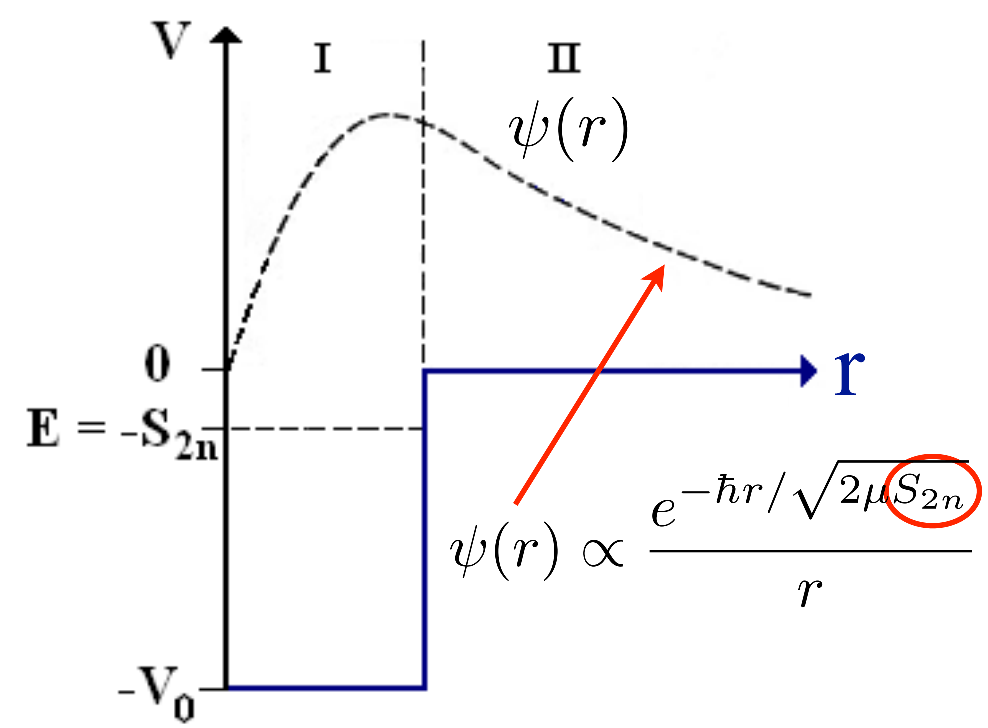


- Tiny one- or two- neutron separation energy

$$S_{xn} = m(Z,N-x) + x\, m_n - m(Z,N)$$







$$\psi(r) \propto \frac{e^{-\hbar r/\sqrt{2\mu S_{2n}}}}{r}$$

- Hansen & Jonson model for ^{11}Li: ^{9}Li core + 2n
- Halo formed by the valence neutrons QM leakage
- S_{2n} regulates extent of halo structure

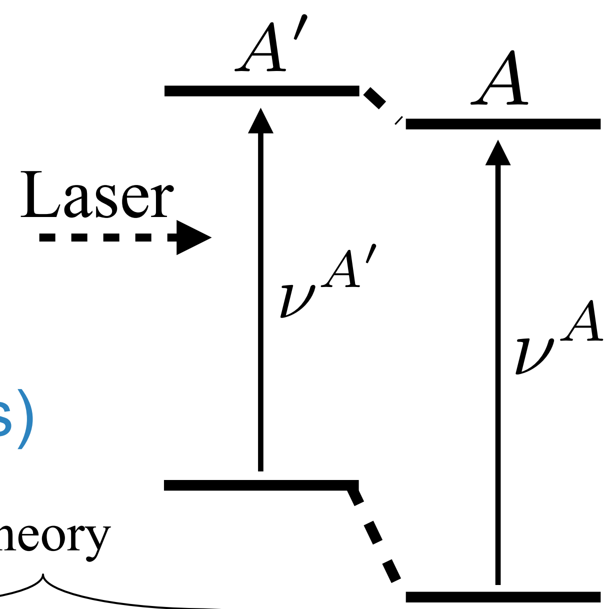
Relevance of the atomic mass

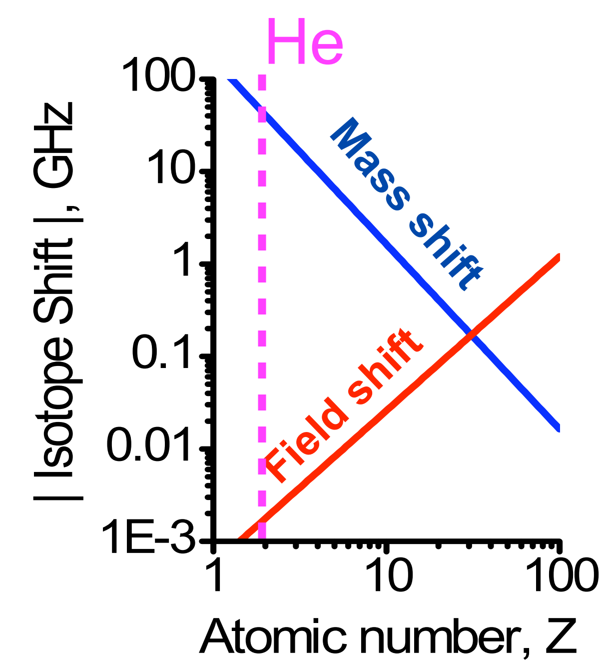
→ The atomic mass is involved in determination of both r_c and S_{2N}

Directly: neutron separation energies

$$S_n = m(Z,N-1) + m_n - m(Z,N)$$

Indirectly: relative charge radius δr_c determination via isotopic shifts measurement (of atomic energy levels)





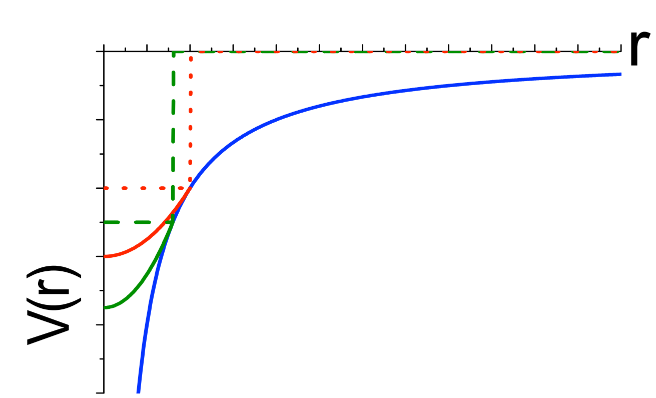
$$\overbrace{\delta\nu^{A,A'} = \nu^{A} - \nu^{A'}}^{\text{Experiment}} = \overbrace{\underbrace{\delta\nu_{MS}^{A,A'}}_{\text{Mass Shift}} + \underbrace{\delta\nu_{FS}^{A,A'}}_{\text{Field Shift}}}^{\text{Theory}}$$

Change in mass of the nucleus $\propto \dfrac{M_A - M_{A'}}{M_A \cdot M_{A'}}$

This change the nucleus recoils

Change in nucleus size $= K_{FS} \cdot \delta\langle r_c^2\rangle^{A,A'}$

This change electrical potential



For ^{8}He, M.S. 72 000 times larger than F.S. and needs to be known at same precision

Importance of mass measurement

→ Mass is the major contribution to the error on ^{8}He relative charge radius

Error budget on relative charge radii

Error	^{6}He (%)	^{8}He(%)
IS Statistical	6	18
Atomic mass	19	58
IS systematics	75	24

Error calculated from: P. Mueller et al., PRL **99,** 252501 (2007)

→ Mass precision required < 350 eV for ^{6}He
< 730 eV for ^{8}He

Tabulated mass excesses (M.E. = m - A)

Isotope	M.E. (keV)
^{6}He	17 595.11 ± 0.76
^{8}He	31 598.0 ± 6.9
^{8}He	31 593 ± 8
^{8}He	31 613 ± 17

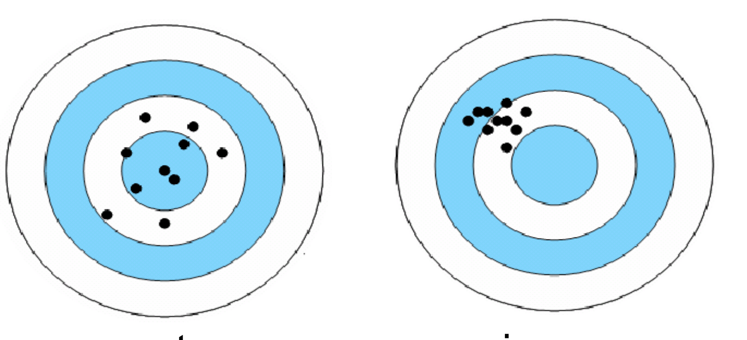
G. Audi et al., NPA **729,** 337 (2003)

← Need 2x more precise
← Need 10x more precise

⇒ relative uncertainty of ~ 1×10^{-7} on the mass needed

The two ^{8}He measurements differs by 20 (19) keV, which could lead to change in relative charge radius of 40%

... masses also need to be more accurate!



accurate, but not precise

precise, but not accurate

Motivation for mass measurement

Besides metrology, why do we need to increase charge radii precision?

→ Current experimental charge radii are at a similar level of precision as theory

→ Need more precise and accurate mass for reliable test of nuclear theory

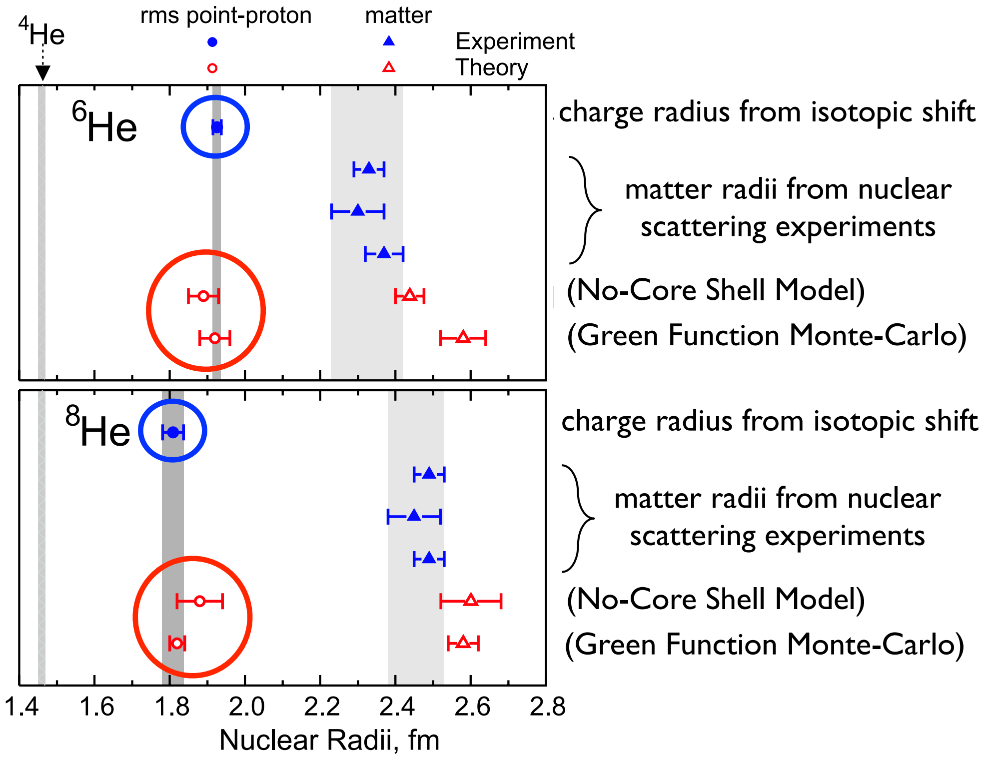


Figure from: P. Mueller et al., PRL **99,** 252501 (2007)

→ from Muller et al., ab-initio theories charge radius predictions for ^{6,8}He agree with value from isotopic shift measurement

→ Does it still hold using the more accurate & precise values obtained from the TITAN masses?

→ How well these methods predicts other observables?

Charge radius (r_C)

Binding energy (E_B)

^{6,8}He halo masses

Two-neutron separation energy (S_{2N})

Unveil systematic deviations in E_B predictions

Check if predicted bound nuclei ($S_{2N} > 0$)

Measurement device

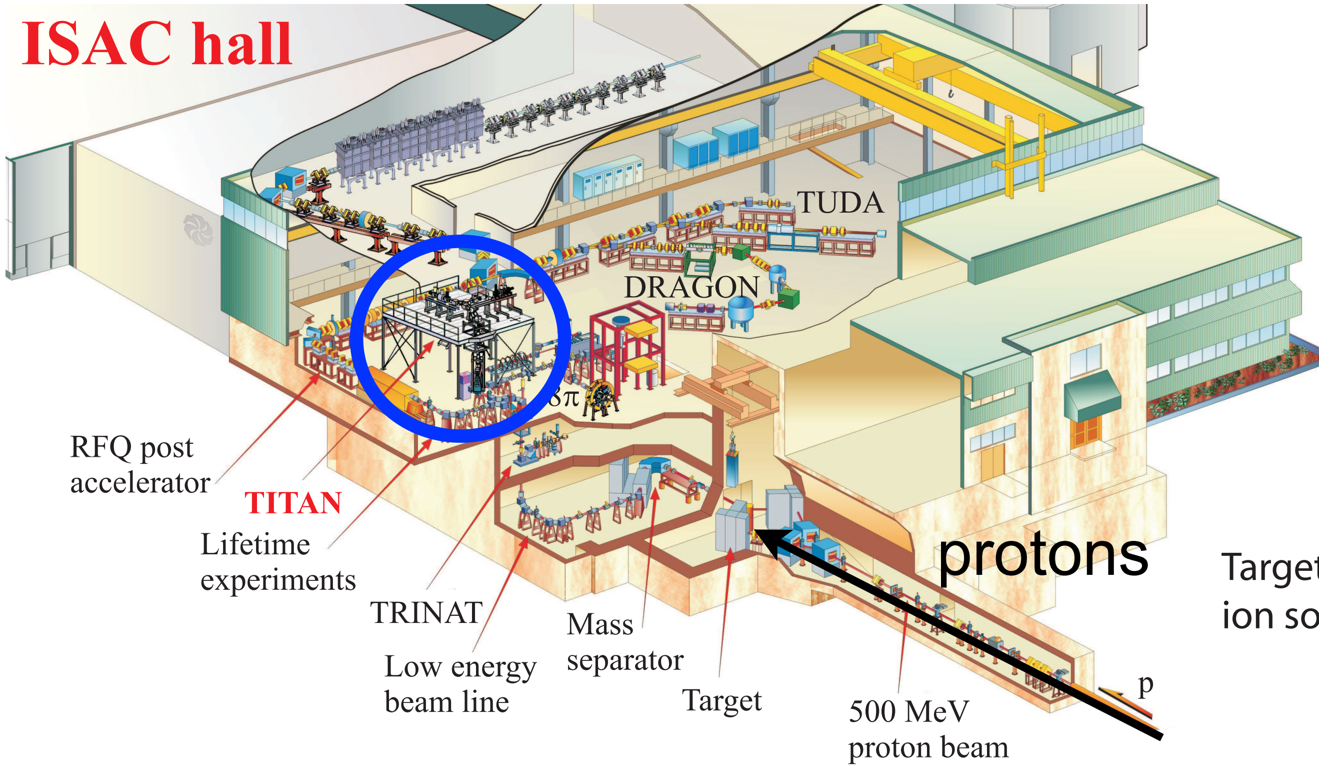
Recall relative uncertainty of $\delta m/m \sim 10^{-7}$
needed for the mass determination of $^{6,8}He$

Only spectrometers that can achieve such precision
and tested to reach this accuracy: Penning traps

Performed measurement with
TITAN Penning trap at TRIUMF

^{6,8}He production at ISAC

Location of the TITAN Penning trap at the TRIUMF ISAC facility



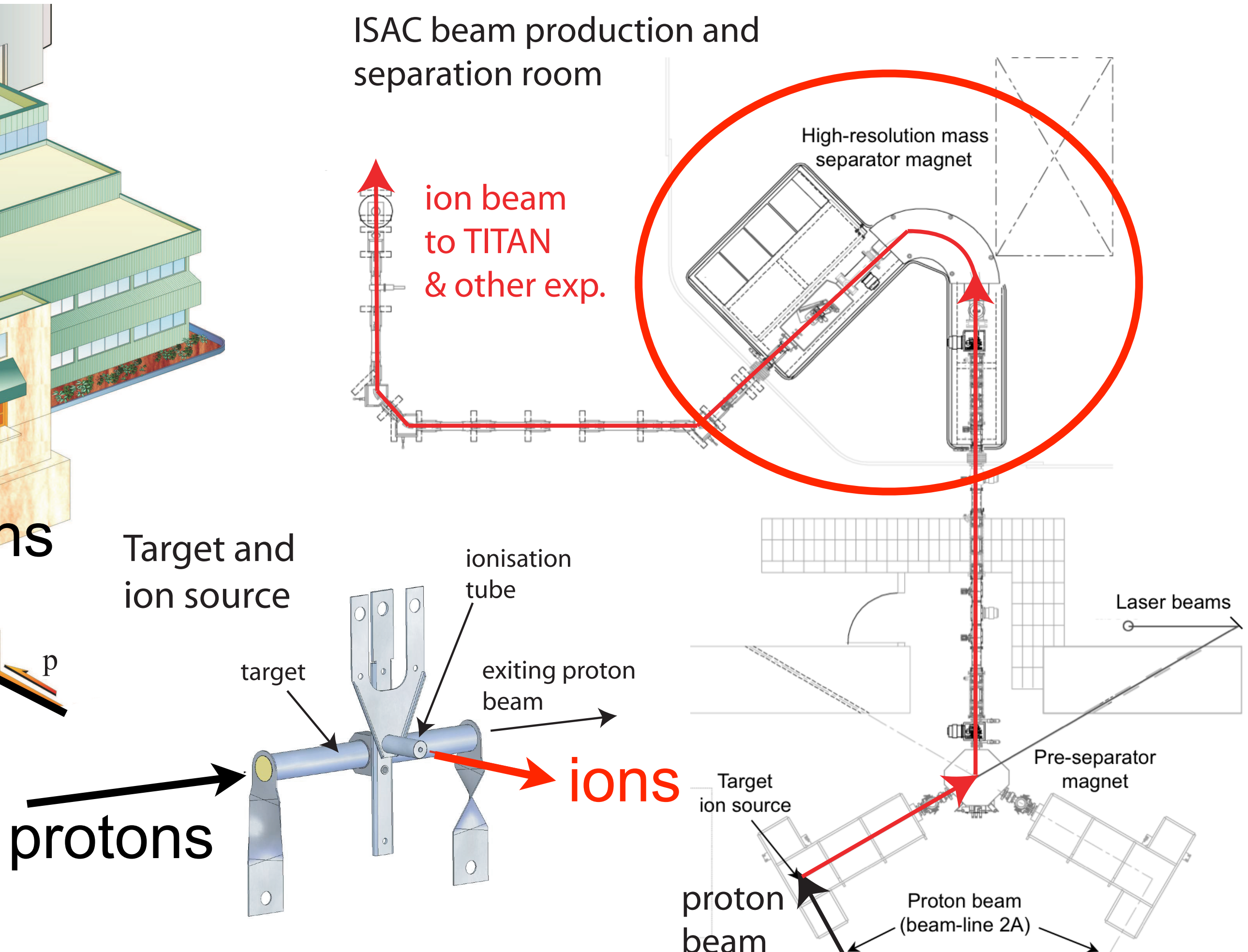


Figure courtesy of J.P. Lavoie

1) 500 MeV protons from cyclotron

2) protons hit thick target, unstable nuclei produced, diffuse, get ionized, extracted

3) contamination removed using mass separator (resolution: $m/\Delta m$ = 3000)

Closest contaminants in mass to ^{6,8}He

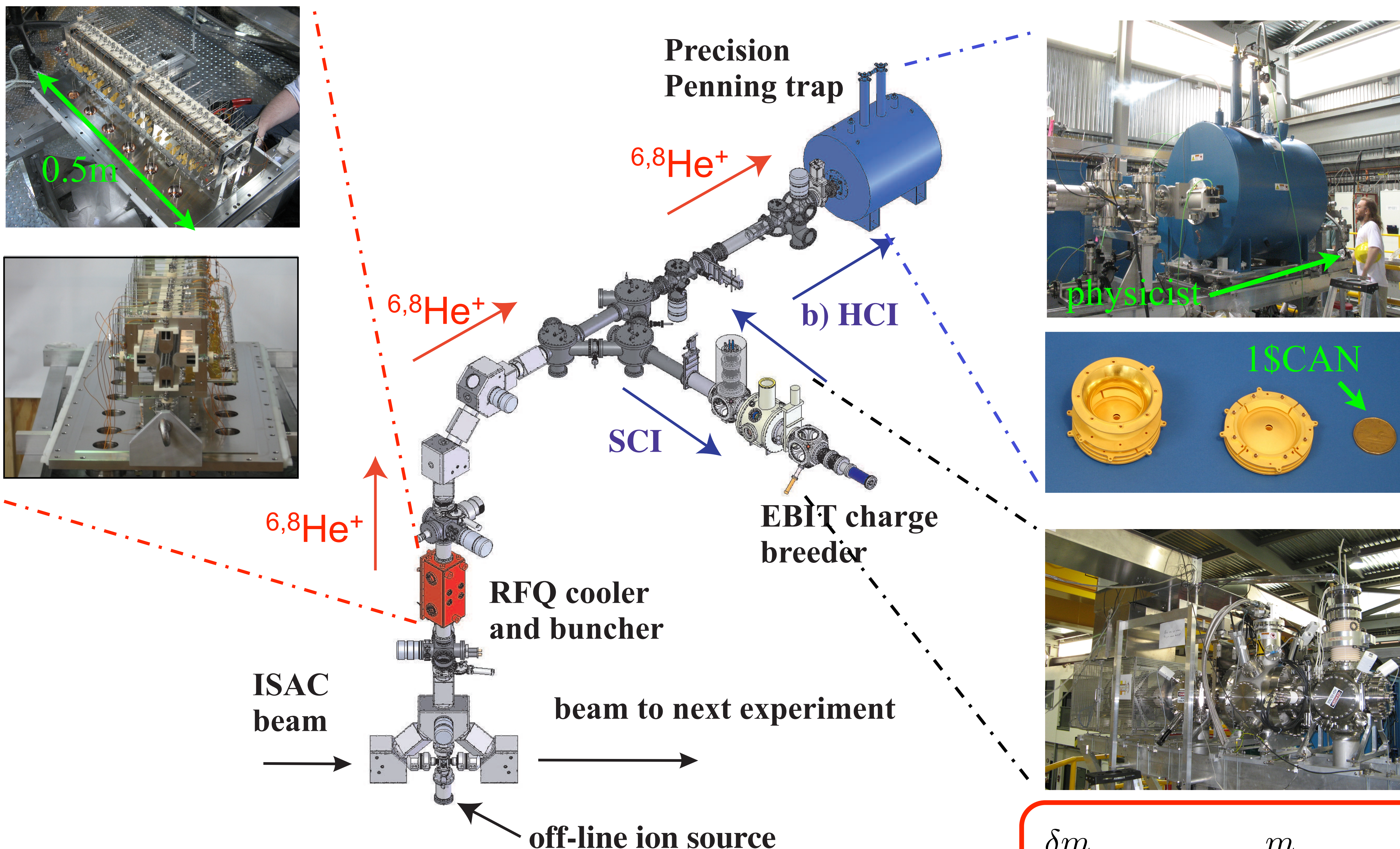
Isotope	Isotope Δ (keV)	contaminant	cont. Δ (keV)	$m/\Delta m$
^{6}He	17 592.09(6)	^{6}Li	14 086.88(2)	1600
^{8}He	31 609.74(12)	^{8}Li	20 945.80(11)	700

(mass excess: $\Delta = m - A$)

→ For the ^{6,8}He mass measurements, the contamination was resolved

TITAN experimental set-up

TITAN: TRIUMF Ion Trap for Atomic and Nuclear science

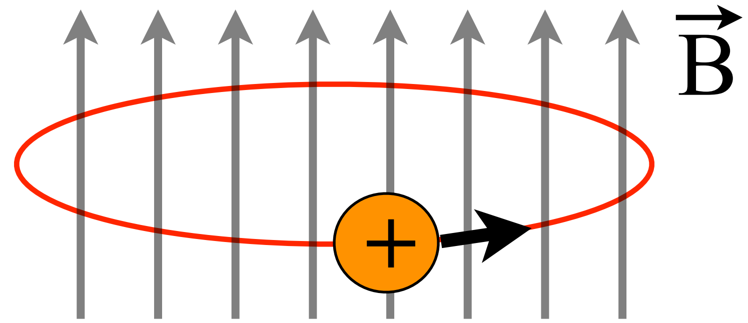


$$\frac{\delta m}{m} \approx \frac{m}{q \cdot B \cdot T_{RF} \cdot \sqrt{N_{ion}}}$$

Penning trap basics

Basic idea:

- Place ion in magnetic field
- Measure cyclotron frequency
- Knowing q & B, get M



$$\nu_c = \frac{1}{2\pi}\frac{q \cdot B}{M}$$

$$\frac{\delta m}{m} \approx \frac{m}{q \cdot B \cdot T_{RF} \cdot \sqrt{N_{ion}}}$$

To achieve high precision (10^{-7}):

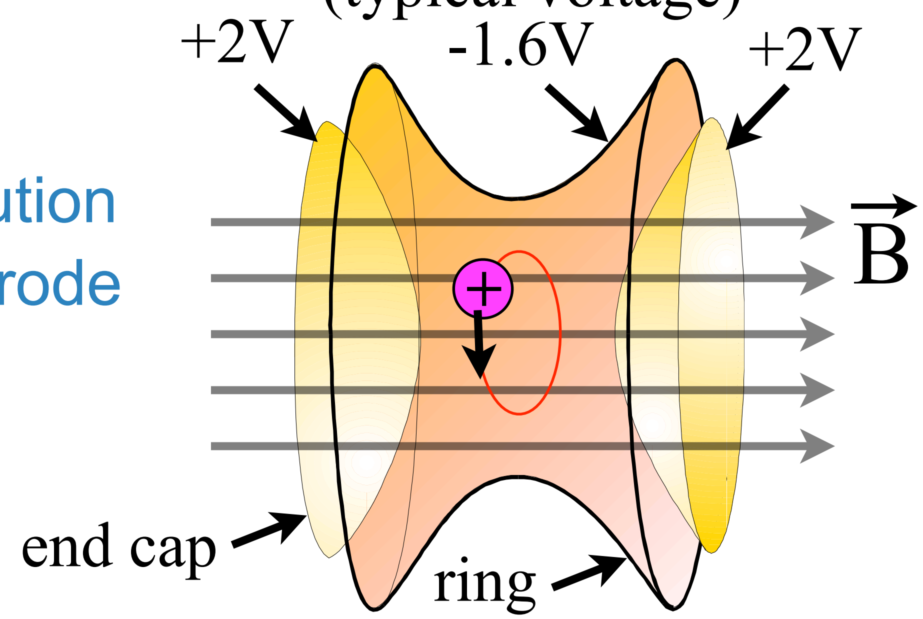
- Need long observation time
- Homogenous B

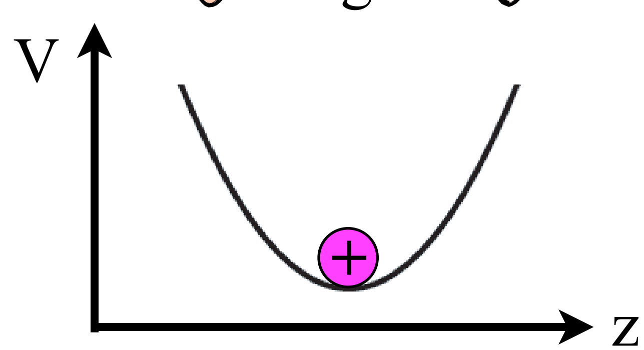
Confine the ions in small volume

Achieved using Penning trap

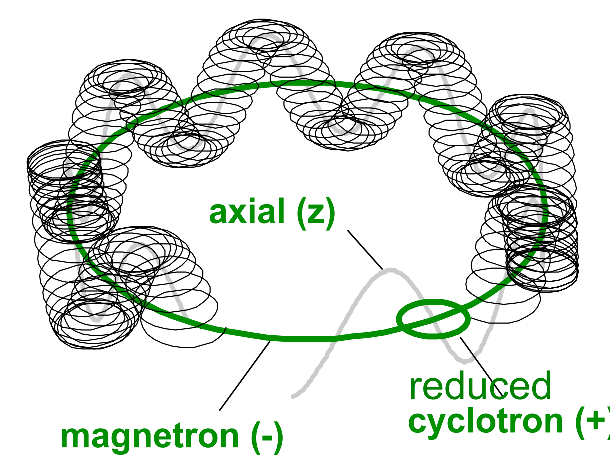
Ideal Penning trap:

- 2 hyperboloids of revolution
  1 ring, 2 end caps electrode
- B: trap radially
- ΔV: trap axially
- Analytical solution for the ion motion





- 3 eigenmotions

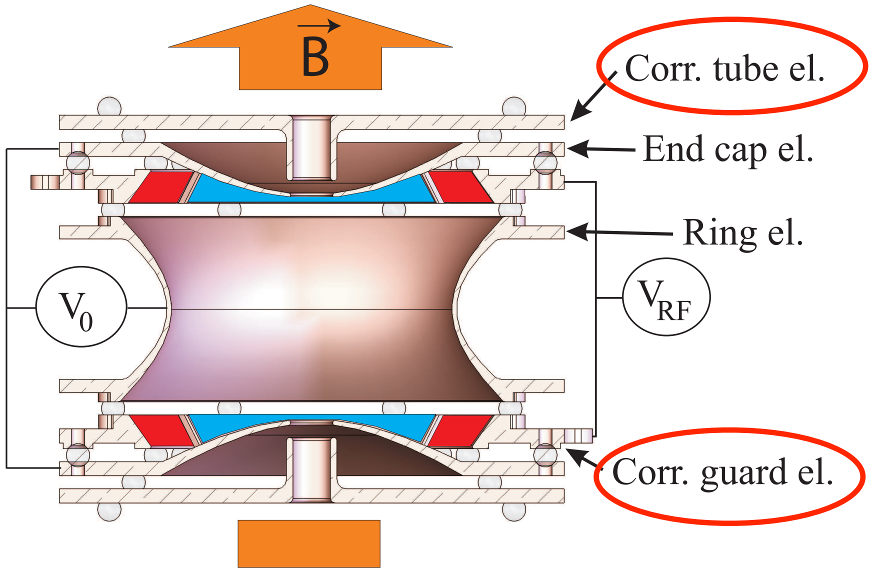


The TITAN Penning trap

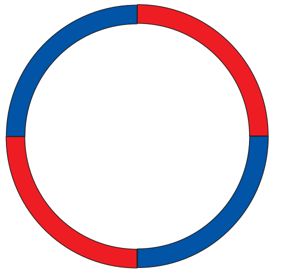
Modifications from ideal trap:
- Hole in end caps (ion insertions)
- Truncation of hyperbola

Correction of non-harm. imperfections
- Tube electrode before end caps
- Guard electrode between end cap and ring

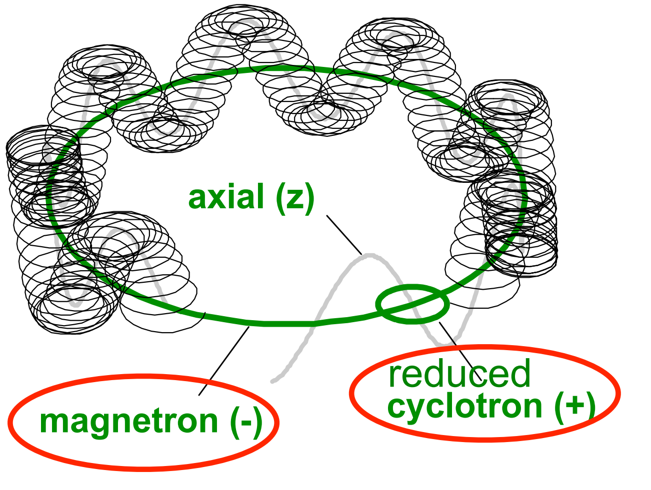
TITAN Penning trap tested to be accurate to 4×10^{-9} using stable species.



Quadrupole excitation at ν_{RF} using guard el.



Effect on ion motion:
- couples the 2 radial motions
- full conversion $\nu_- \rightarrow \nu_+$ when $\nu_{RF} = \nu_c = \nu_- + \nu_+$



The Time-Of-Flight Ion Cyclotron Resonance (TOF-ICR) technique:
- Cyclotron frequency ν_c determined by finding ν_{RF} for which full magnetron to reduced cyclotron conversion is achieved

TOF-ICR technique

(M. Konig et al., Int. J. Mass Spectr. Ion. Proc., 142, 95 (1995))

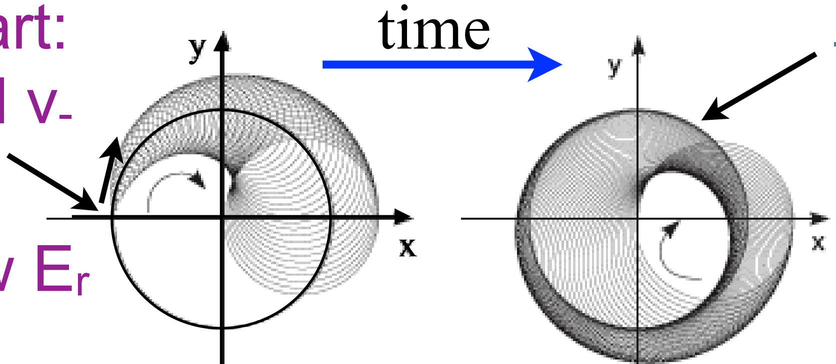
TOF-ICR Determination of the cyclotron frequency

1) Prepare the ions in full magnetron motion

2) Excite with quadrupole freq. ν_{RF} for time T_{RF}

start: full ν_-

low E_r



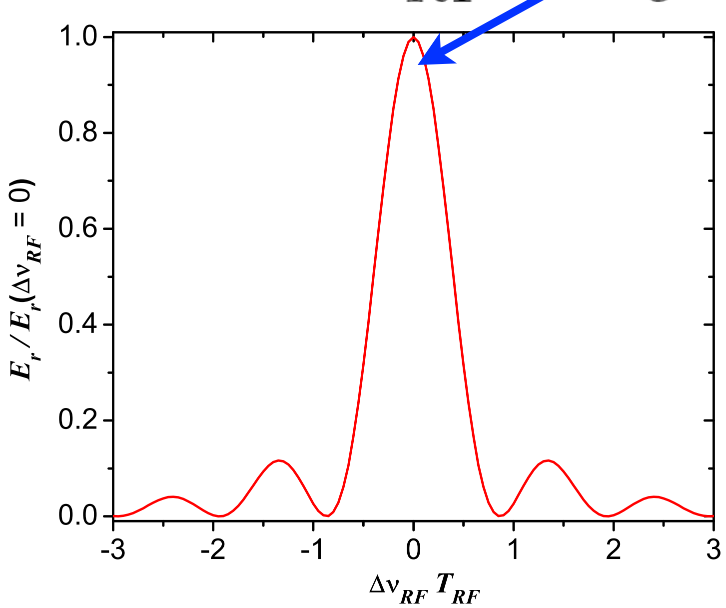
finish: full ν_+ if $\nu_{RF} = \nu_c$

high E_r

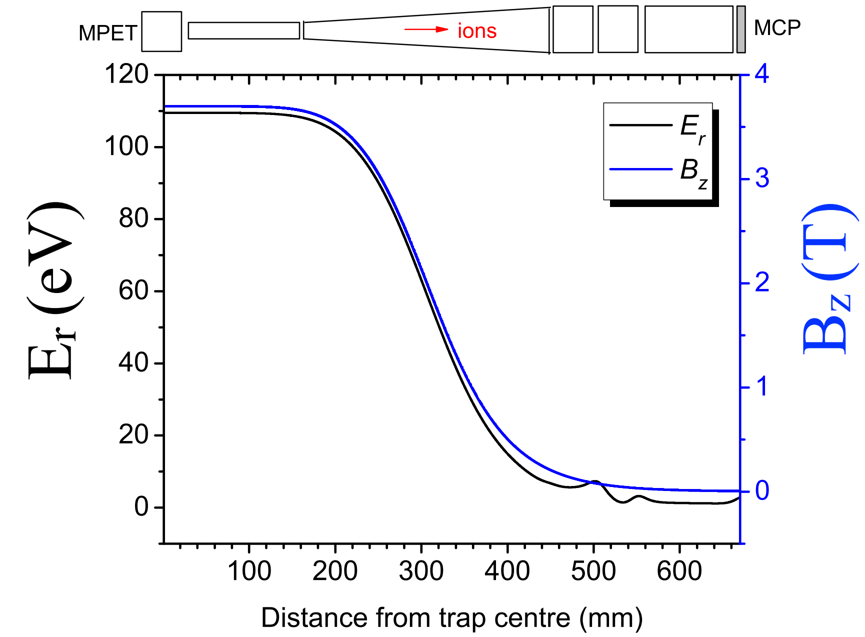
$^6Li^+$ frequencies

ν_c	9.451 MHz
ν_+	9.445 MHz
ν_z	339 kHz
ν_-	6.1 kHz

Max E_r when $\nu_{RF} = \nu_c$



3) Ion release from trap and ν_c determined from time-of-flight

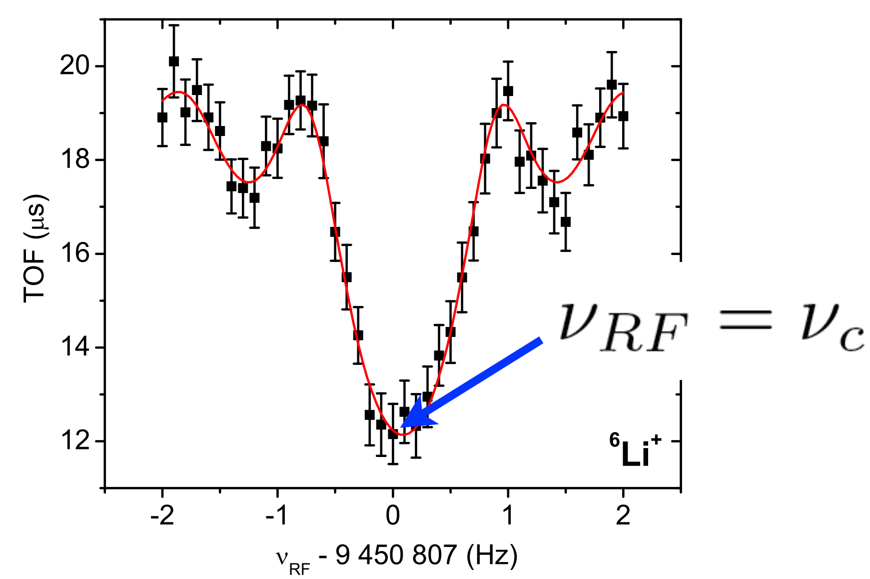


→ force on the ions:

$$\vec{F} = -\vec{\nabla}U = \vec{\nabla}(\vec{\mu}\cdot\vec{B}) = -\frac{E_r}{B_0}\frac{\partial B_z}{\partial z}\hat{z}$$

→ ions with $\nu_{RF} = \nu_c$ arrive sooner

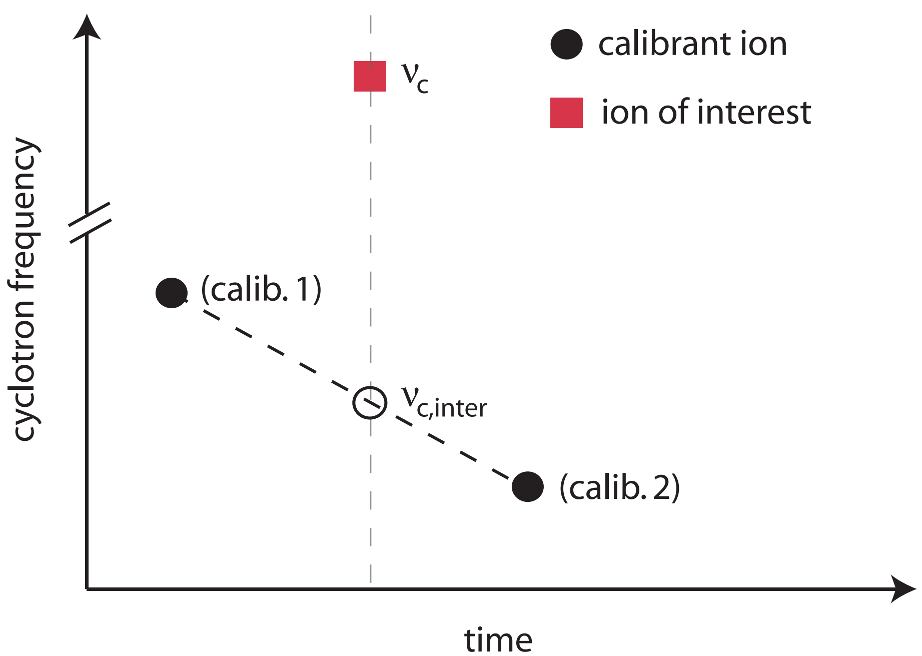
→ ν_c determined by stepping ν_{RF}



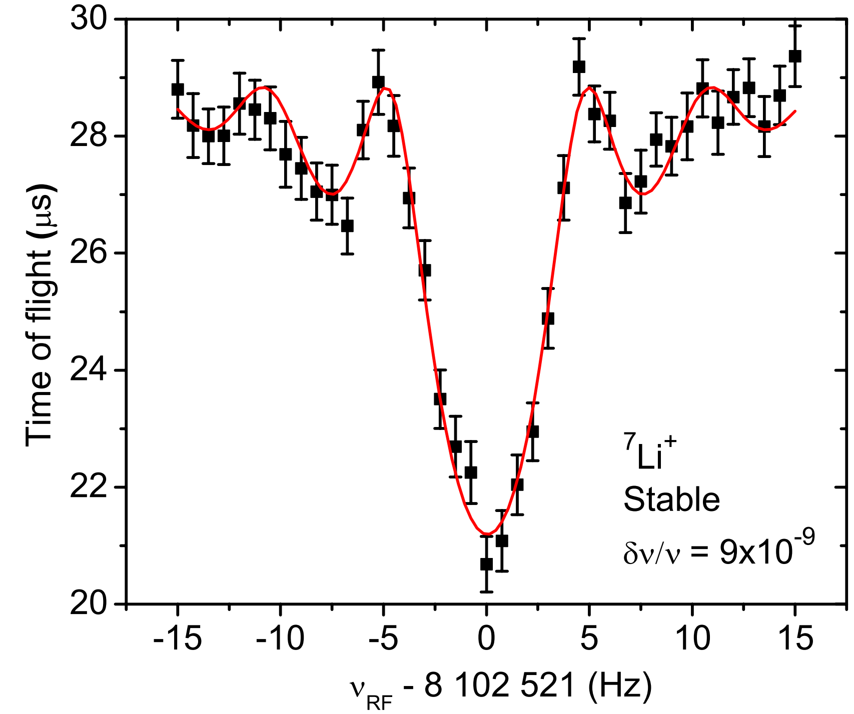
Mass determination

Measurement: $\nu_c = \frac{1}{2\pi}\frac{q \cdot B}{M}$

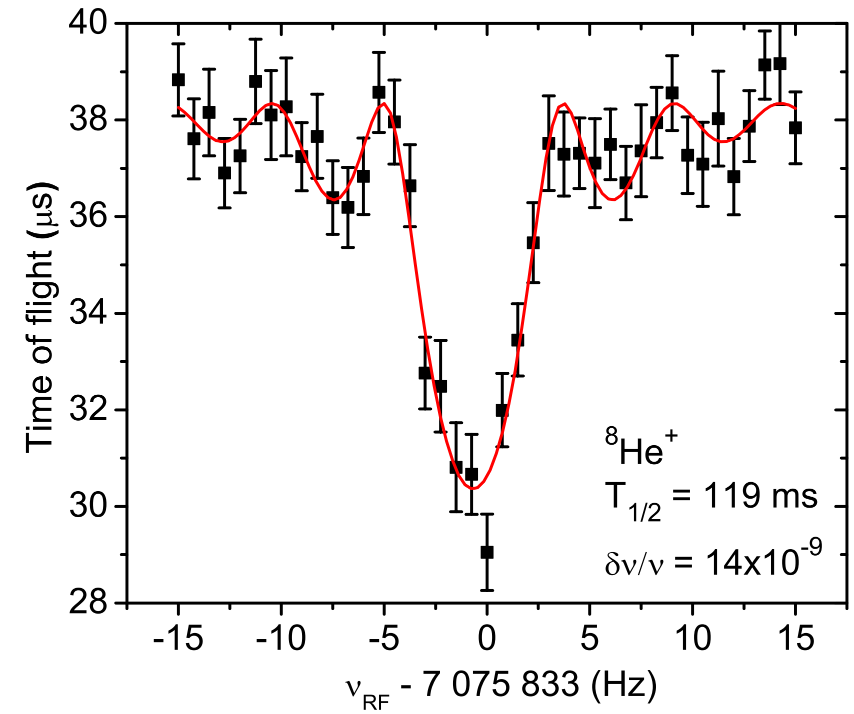
ion's mass



calibrant ion



ion of interest



Experimental result: $R = \nu_{c,inter}/\nu_c$

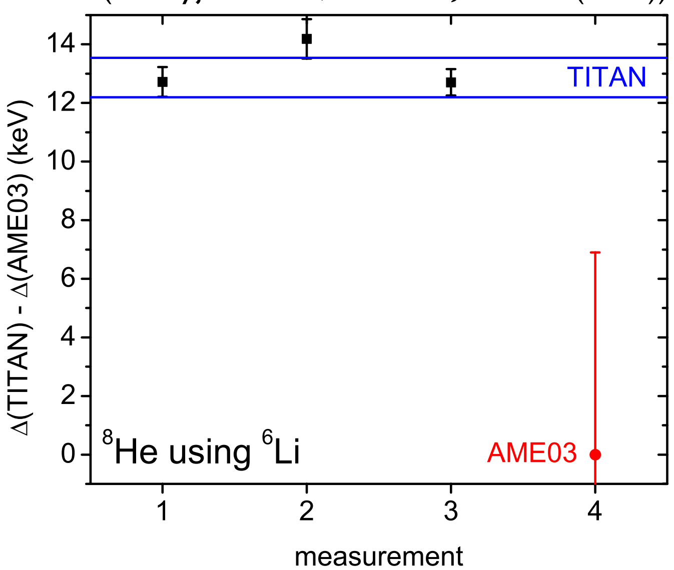
Atomic mass:

$$m = \overline{R} \cdot (m_{cal} - m_e + B_{e,cal}) + m_e - B_e$$

^{6}He & ^{8}He mass measurements

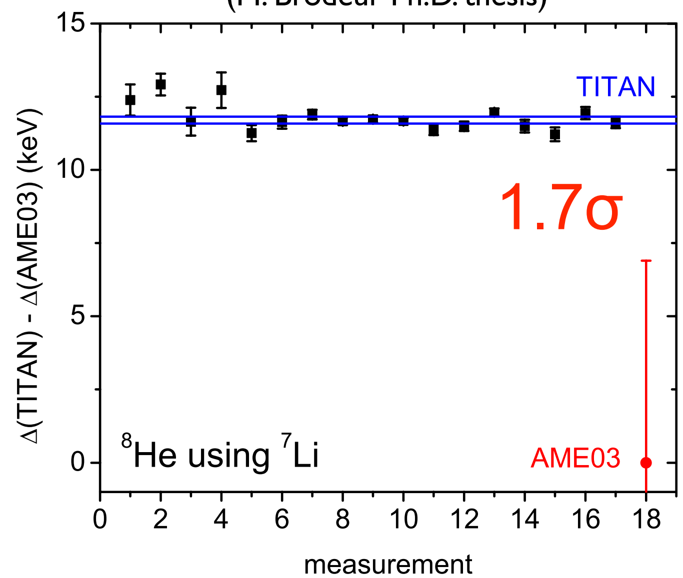
TITAN mass excesses Δ compared to AME03

1st ^{8}He mass meas.
(V.L. Ryjkov et al., PRL **101**, 012501 (2008))



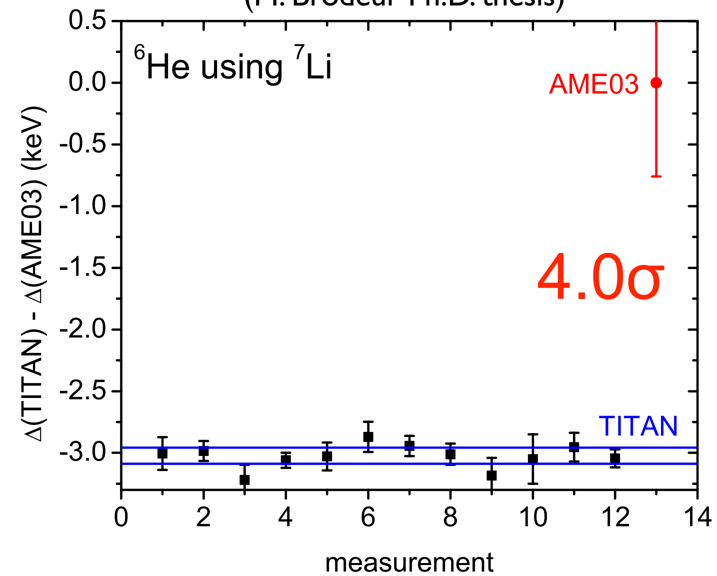
counts rate: ~3 ions/min

2nd ^{8}He mass meas.
(M. Brodeur Ph.D. thesis)



~40 ions/min

^{6}He mass meas.
(M. Brodeur Ph.D. thesis)



~100 ions/min

Error budget (note: trapping potential V_0 = 3.6V)

Error	$\Delta R/R \times 10^{-9}$ (^{6}He)	$\Delta R/R \times 10^{-9}$ (^{8}He)
Statistical	4.9	5.9
Ion-ion interaction	8.1	13.3
Total	9.4	14.6

Upper limit on the error due to the interaction between ^{6,8}He and ionized background gas.

(conservative estimate obtained from count rate analysis)

The other sources of systematic errors are < 1 ppb

Accuracy check: mass measurement of ^{6}Li and ^{4}He

Isotope	Δ(TITAN) (keV)	Δ(lit.) (keV)	δΔ (eV)
^{4}He	2 424.915(18)	2 424.915 65(6)	-1(18)
^{6}Li	14 086.867(9)	14 086.881(20)	-14(22)

Both results agrees with literature

Results and discussion

New binding and 2n separation energies:

$$E_B(N,Z) = m(N,Z) - Zm_H - Nm_n$$

$$S_{2N}(Z,N) = m(Z,N-2) + 2m_n - m(Z,N)$$

Isotope	E_B (keV)	S_{2N} (keV)
^{6}He	-29 271.123(76)	975.46(24)
^{8}He	-31 396.134(133)	2125.01(37)

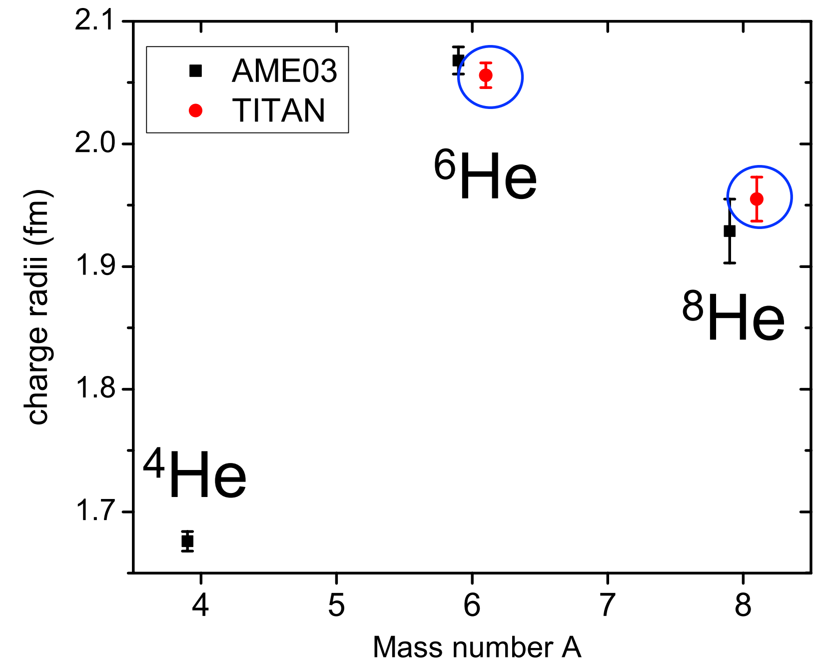
New charge radii:

$$\langle r_c^2 \rangle^A = \langle r_c^2 \rangle^4 + \frac{\delta\nu^{A,4} - \delta\nu^{A,4}_{MS}}{K_{FS}}$$

Isotope	$\langle r^2 \rangle_c^{1/2}$ (AME03)	$\langle r^2 \rangle_c^{1/2}$ (TITAN)	$\langle r^2 \rangle_{pp}^{1/2}$ (TITAN)
^{6}He	2.068(11)	2.056(10)	1.913(9)
^{8}He	1.929(26)	1.955(17)	1.835(18)

36% precision improvement

Both charge radii changed, difference in charge radii is 0.04(3) fm smaller



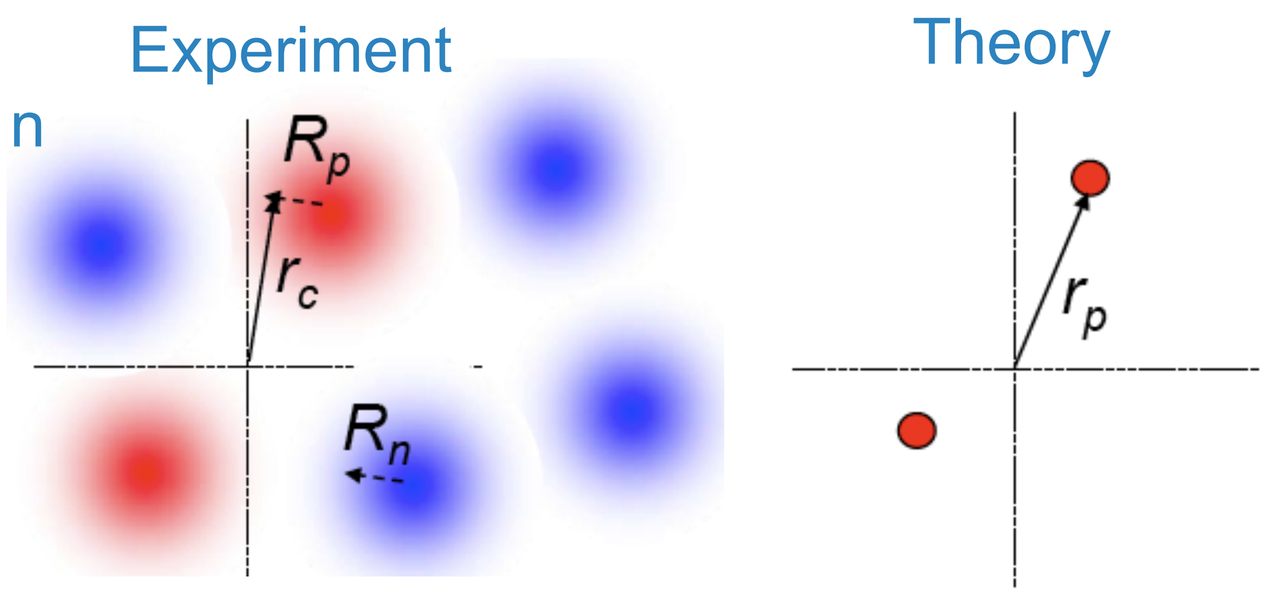
Point-proton radii:

The measured r_c includes the size of p & n

Theory assumes point-particle

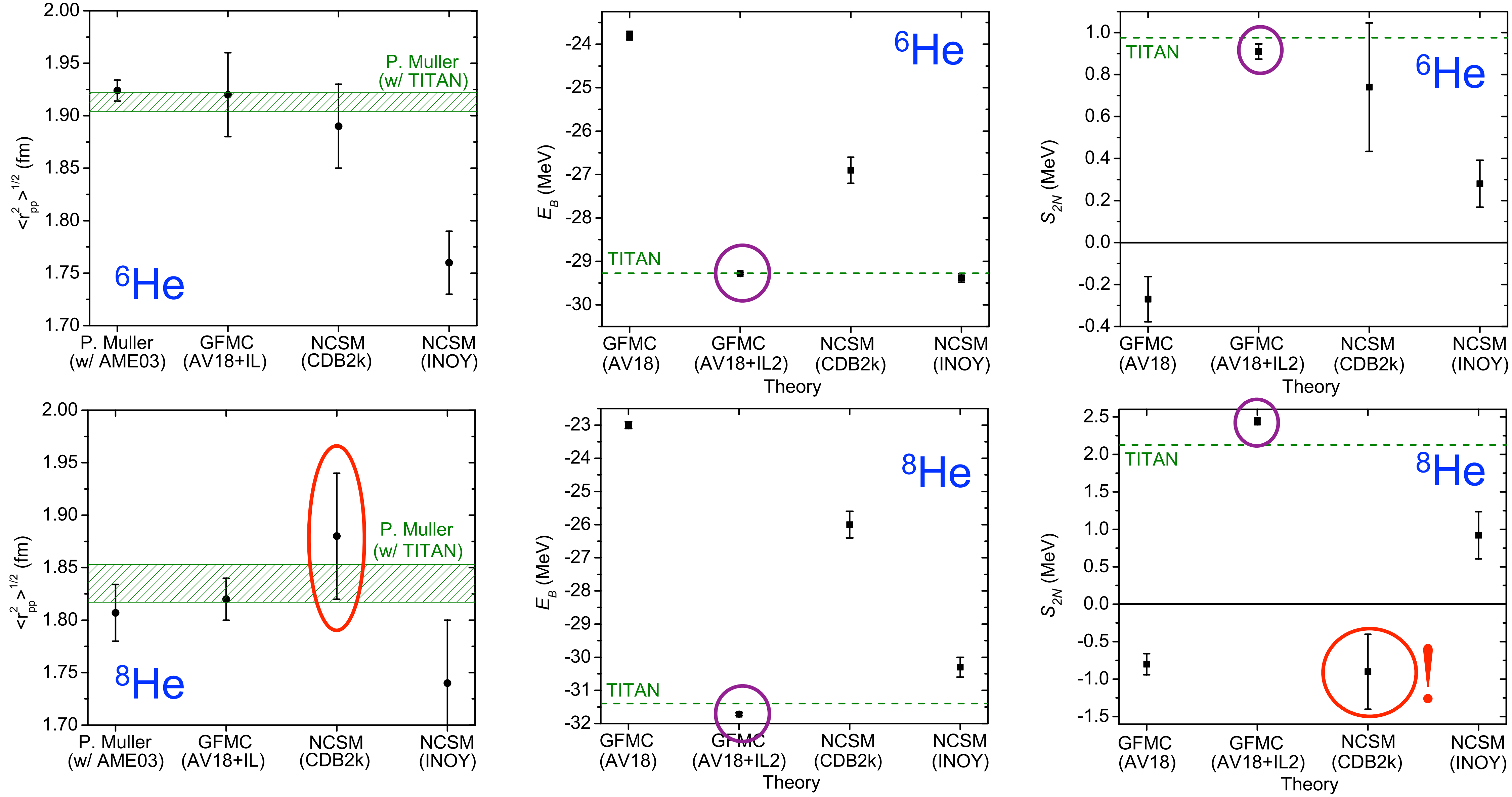
Need to correct measured r_c in order to compare with theory

$$\langle r^2 \rangle_{pp} = \langle r^2 \rangle_c - \langle R_p^2 \rangle - \frac{N}{Z}\langle R_n^2 \rangle - \frac{3}{4M_p^2}$$



Comparison with theory

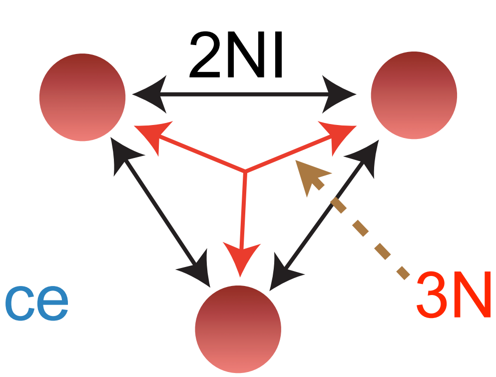
Charge radius — Binding energy — 2n separation energy



- Both the GFMC & NCSM r_c agrees with new exp. ^{6,8}He r_c

GFMC (AV18+IL) → Method that provides the closest values to experiment
Only method that uses 3 nucleons interaction (3NI)

NCSM (CDB2k) → Produce a physical r_c for an unbound nuclei, consequence of using faster Gaussian fall-off and small model space.



Summary & Outlook

- Found deviations of 3.02 and 11.67 keV for the respective ^{6,8}He masses compared to tabulated values
- The uncertainties on the new charges radii are now independent of the atomic mass
- We showed that using 2 observables involving the mass of halo nuclei, we can test the limitations of ab-initio methods

- There are more halo mass measurements to come at TITAN, including ^{14}Be and ^{19}C.
- Other mass measurements are planned at TITAN, including neutron-rich K and Ca to study change in the nuclear structure

Acknowledgements

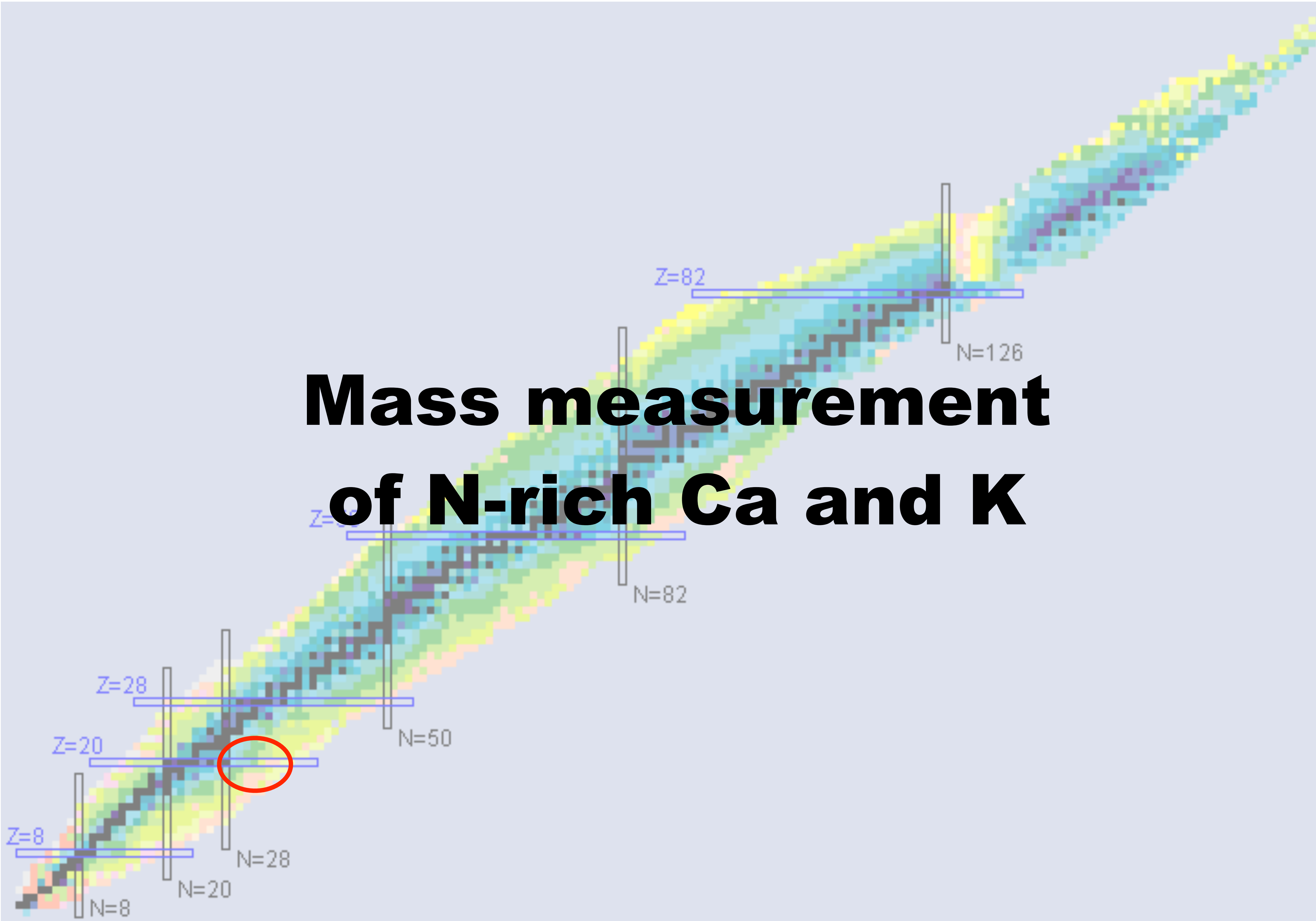
- The TITAN Group: Jens Dilling, Paul Delheij, Gerald Gwinner, Melvin Good, David Lunney, Mathew Pearson, Alain Lapierre, Ernesto Mané, Ryan Ringle, Vladimir Ryjkov, Thomas Brunner, Stephan Ettenauer, Aaron Gallant, Vanessa Simon, Mathew Smith
- TRIUMF Staff: Pierre Bricault, Ames Freidhelm, Jens Lassen, Marik Dombsky, Rolf Kietel, Don Dale, Hubert Hui, Kevin Langton, Mike McDonald, Raymond Dubé, Tim Stanford, Stuart Austin, Zlatko Bjelic, Daniel Rowbotham, Daryl Bishop
- TRIUMF Theory Group: Sonia Bacca, Achim Schwenk
- Special thanks: Gordon Drake

And the rest of the TITAN collaboration....

U. of Manitoba
McGill U.
Muenster U.,
Max Plank Inst. of Heidelberg
GANIL
CNRS-CSNSM
U. of Windsor
Colorado School of Mines
TRIUMF
UBC

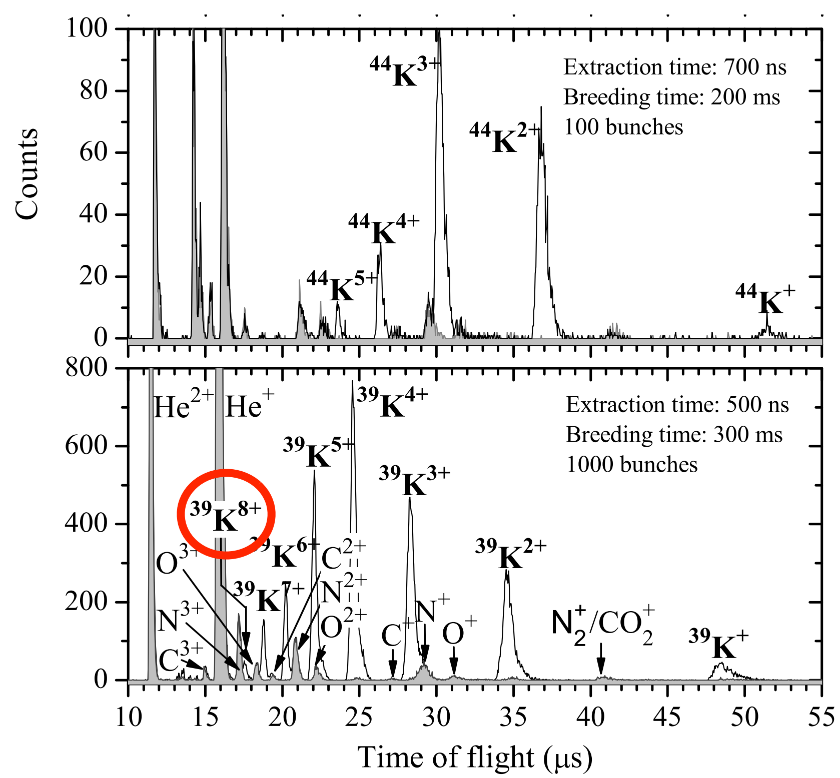
Back-up slides

Mass measurement of N-rich Ca and K



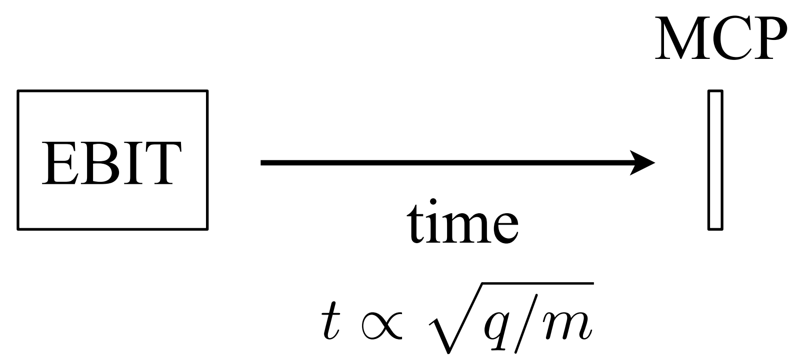
Charge-bred radioactive ion beam

Time-of-flight distribution of charge bred ions from the TITAN EBIT



injected beam: potassium

rest: charge bred residual gas

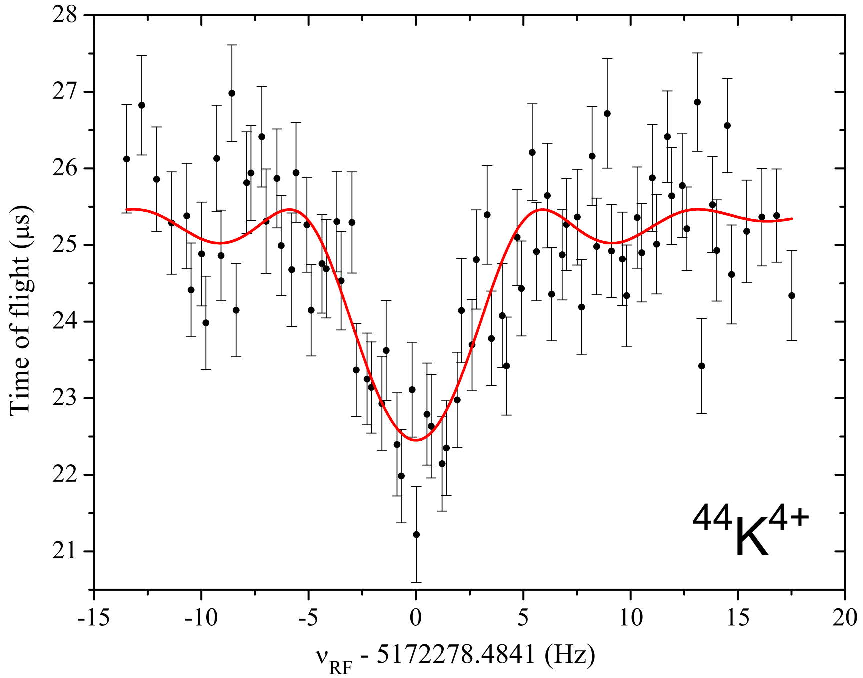


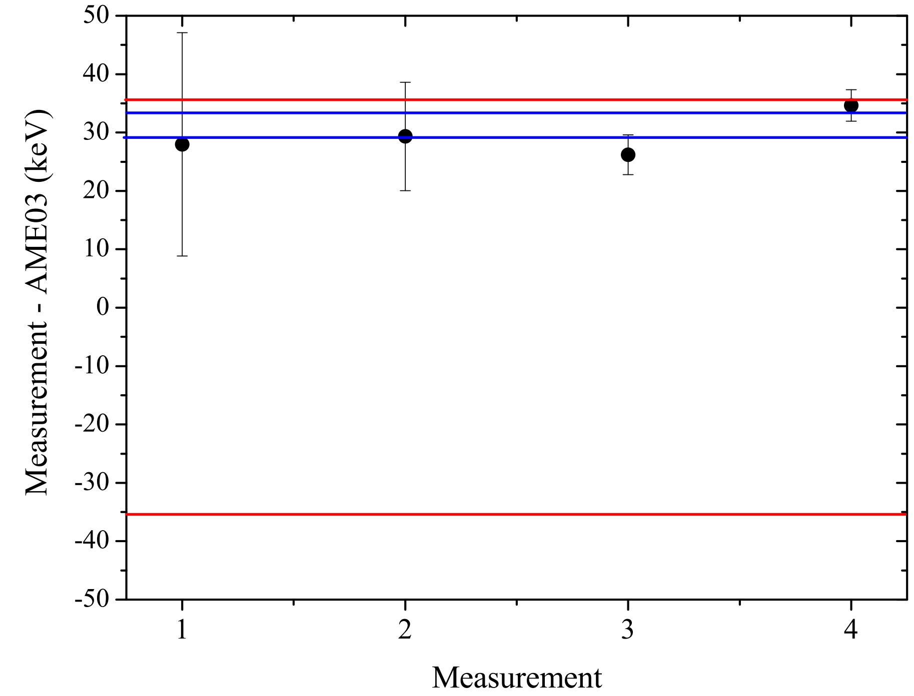
- Observed up to 8+ charge state of ^{39}K for 2 keV e-beam energy
- Charge breeding of the residual gas (O_2, N_2, H_2) and ^{4}He from RFQ makes it presently difficult to use higher charge states of injected ions for the Penning trap.
- Total efficiency for injection/charge breeding and extraction of $^{44}K^{4+}$: 0.1%
- Charge state 4+ is not the dominant one, but the easiest to resolve from residual gas contamination

First charge-bred RIB mass meas.

A. Lapierre et. al, in preparation

First Penning trap mass measurement using charge-bred ions: $^{44}K^{4+}$





Factor of 10 improvement on the AME03 mass

Future work needed for mass measurements using HCIs:

1) Improve EBIT efficiency for HCI production/transport

Plans: evaporative cooling in the EBIT

cooling using the cooling Penning trap (as discussed by V. Simon)

dipole cleaning in the EBIT (already demonstrated)

charge state ratio optimization

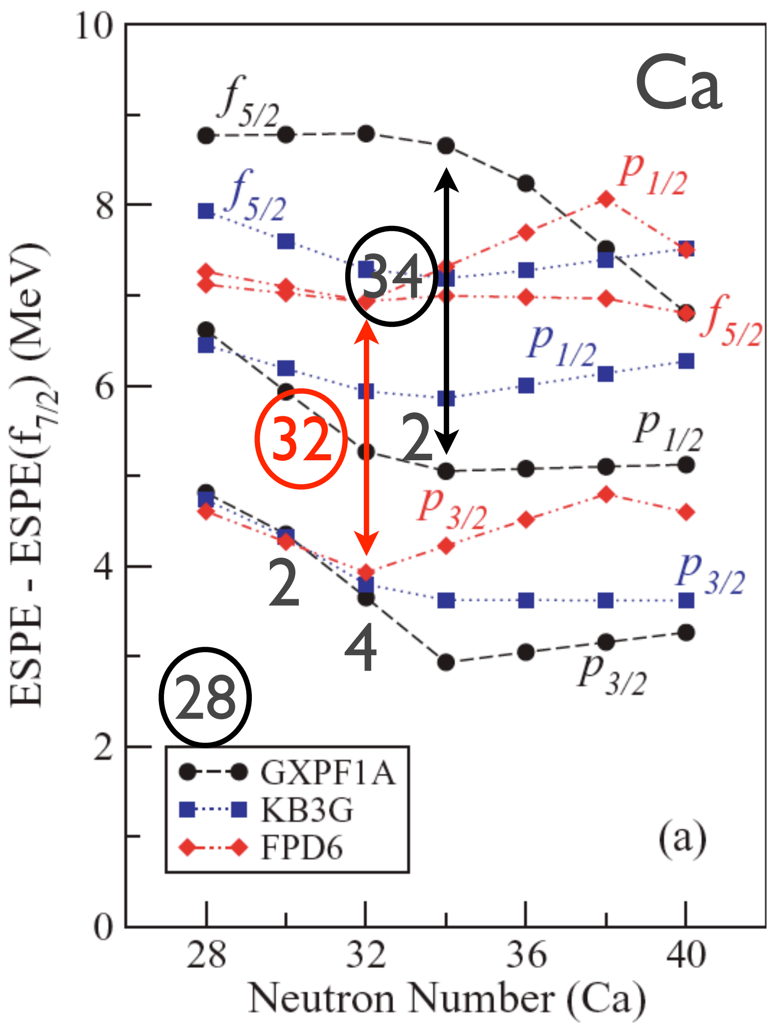
2) vacuum in the Penning trap

→ Excitation time limited to 200 ms due to Penning trap vacuum

Solution: baked the Penning trap (now we reached 4 x 10^{-11} torr)

Motivations for the measurement

As an element gets more N-rich, its shell structure changes.
This induce a change in the magic numbers



(Effective Single Particle Energy)
(from S.Zhu, PRC **80**, 024318 (2009)

- The various existing nuclear models predicts different new magic numbers for Ca

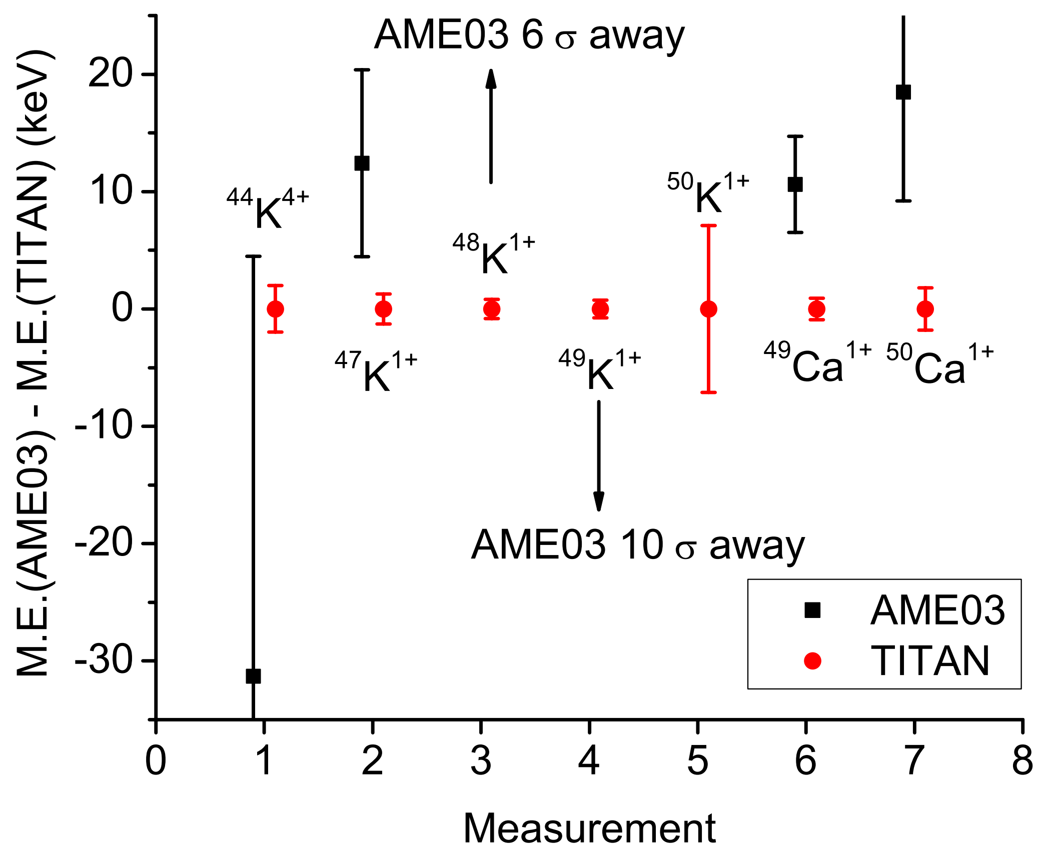
FPD6: N = 32 (Analytic 2-body pot.; selected energy levels)

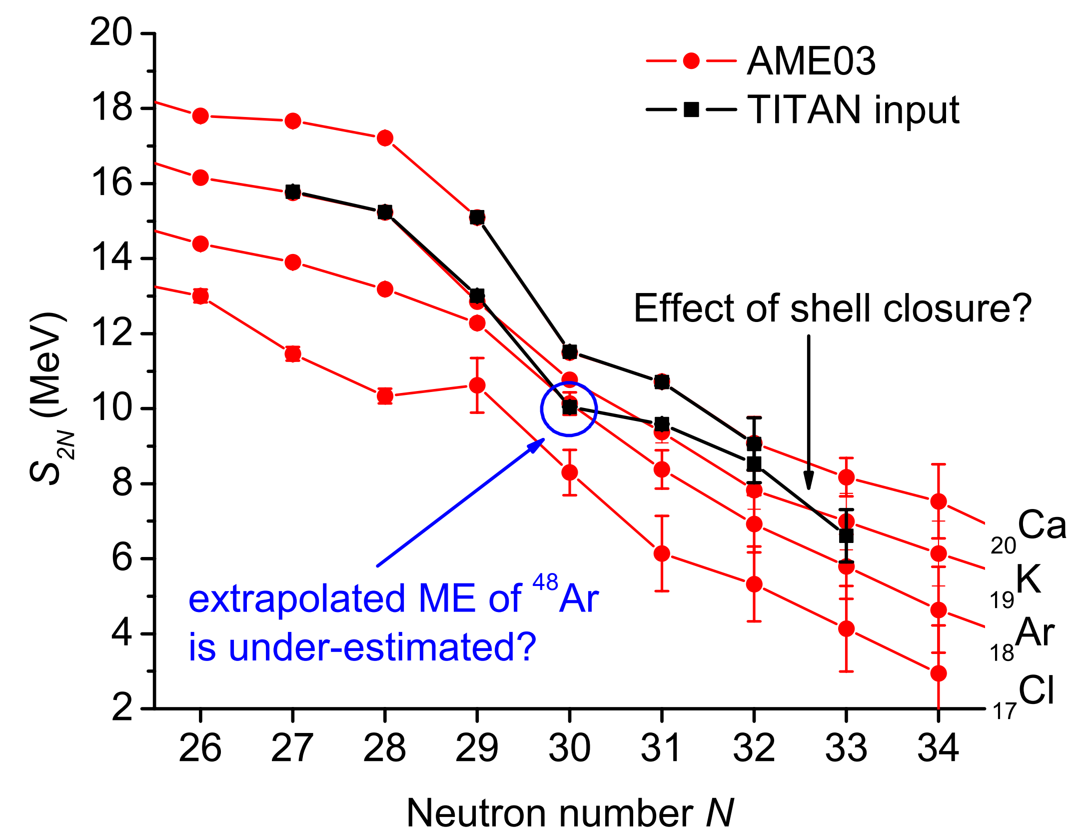
GXPF1A: N = 34 (G-matrix pot.; full fp shell; cross-shell exc.)

KB3G: no new (Kuo-Brown G-matrix pot.; full fp shell)

- Goal: put tighter constrain on nuclear models predictions through mass measurement
- As the above models only include 2-body forces, 3-body forces might be required to explain our findings
- New magic numbers were previously found, such as ^{24}O

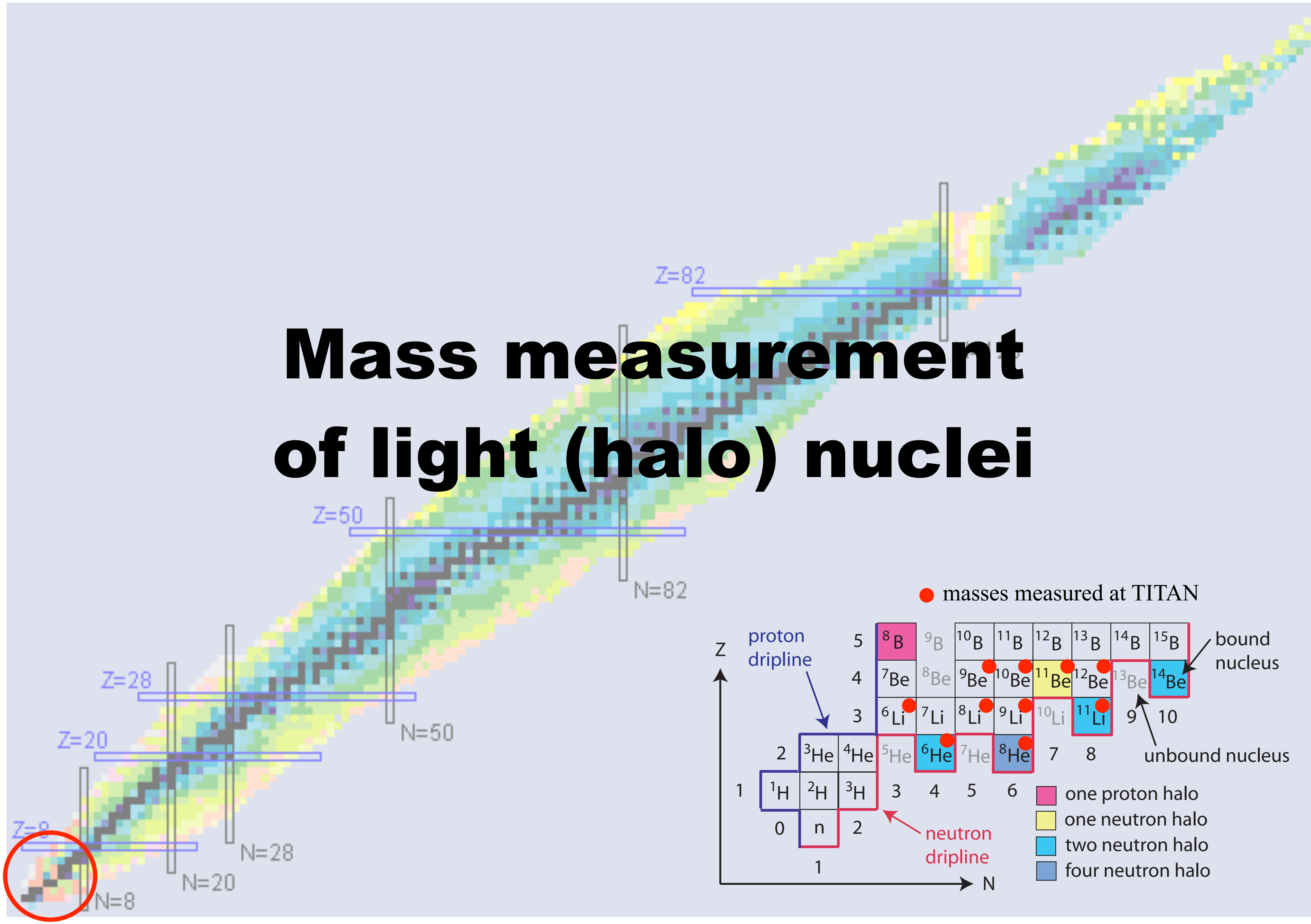
Mass measurements on K & Ca





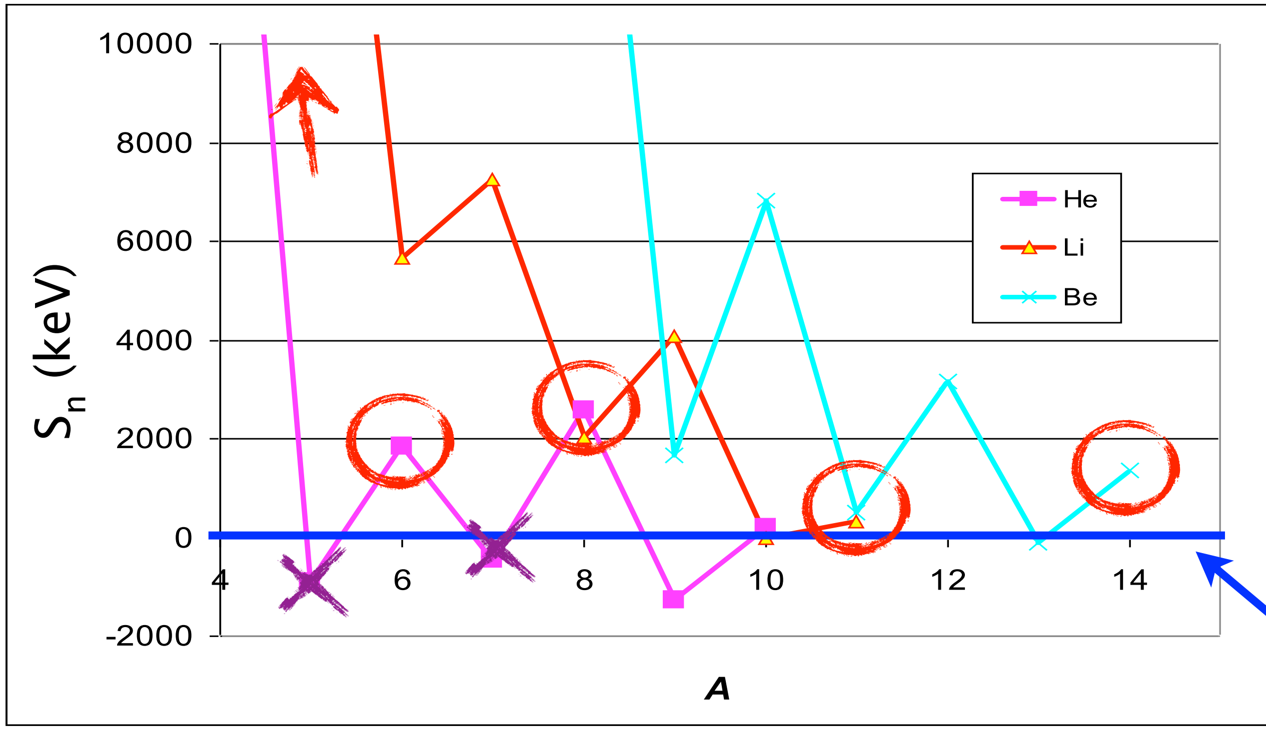
- $^{47\text{-}50}$K and ^{49,50}Ca masses improved by factor of up to 100
- ^{48}K and ^{49}K masses deviates by 6 and 10 σ from AME03
- ^{51}K and ^{52}K mass measurement needed to see if shell closure at N = 32
- $S_{2N}(^{51}K)$ ~ $S_{2N}(^{52}K)$: extrapolated ^{48}Ar mass could be under-estimated mass measurement of ^{46}Ar and ^{48}Ar are needed...
- As the N-rich mass landscape gets more refined, more measurements are needed!

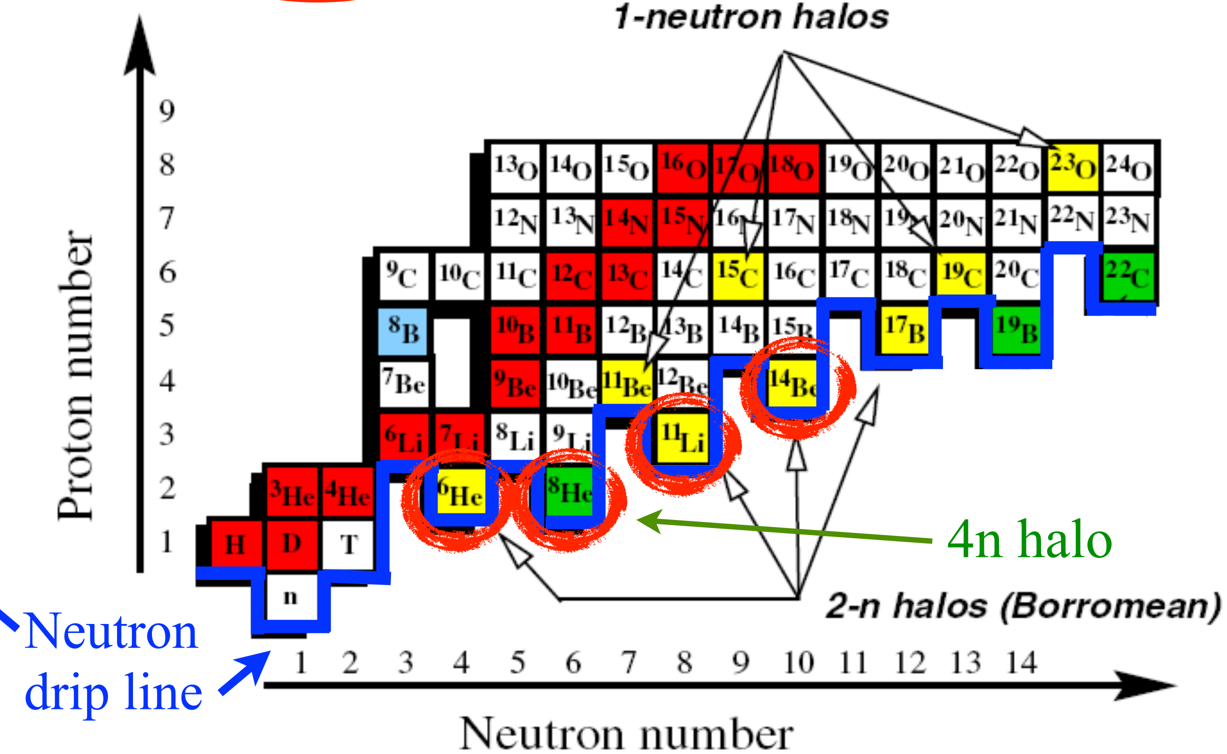
Mass measurement of light (halo) nuclei

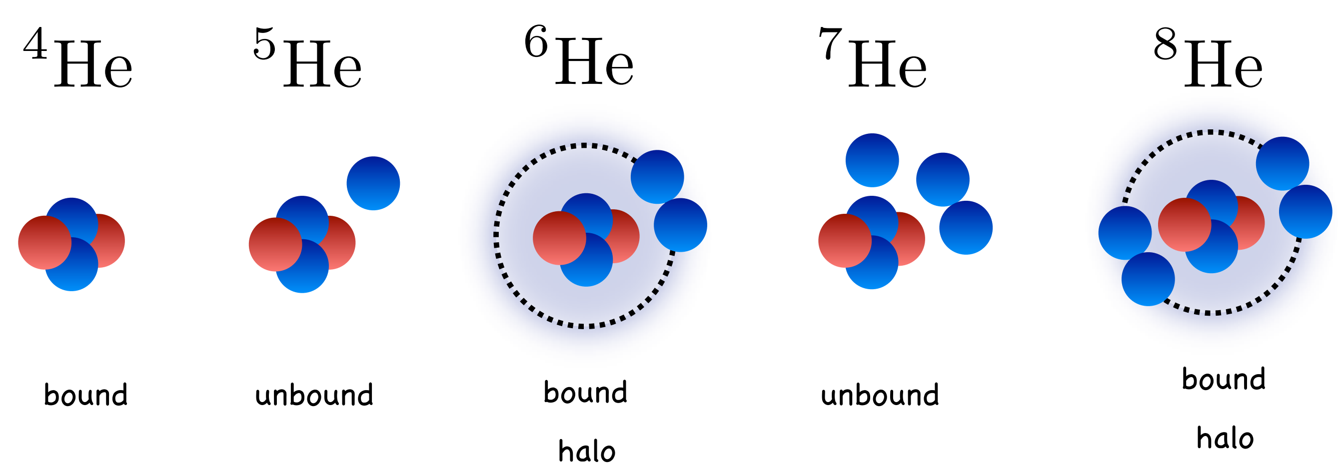


The role of mass: separation energy

One neutron separation energy: $S_n(N,Z) = m(N-1,Z) + m_n - m(N,Z)$

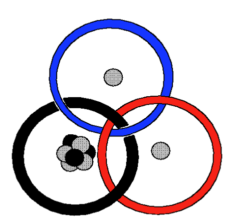






Halos are a threshold phenomenon

Borromean halo:



n + n: ^{2}n unbound

^{4}He + n: ^{5}He unbound

^{4}He + n + n: ^{6}He bound

Neutron separation energy: key quantity to characterize halo

Directly involves the atomic mass

Charge radius definition

The electrostatic potential energy of the atomic system can be express as: $U = \int \rho(\vec{r}) V(\vec{r}) d^3\vec{r}$

The atomic nucleus is small enough compared to the size of the atom, than one can define a radius R such that $\rho(\vec{r}) = 0$ if r < R.

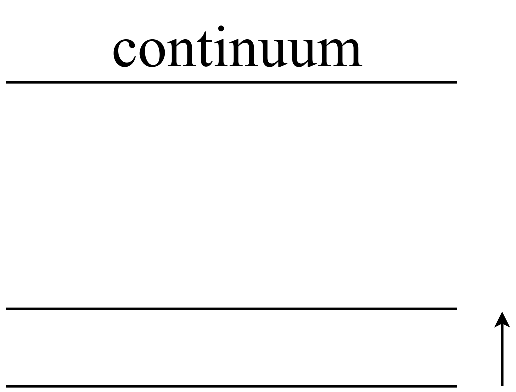
For r < R, the potential assuming constant electron density is: $V(r) = V(0) + \frac{e}{2\epsilon_0} \cdot |\psi(0)|^2 \cdot \frac{r^2}{3}$

This gives:

$$U = V(0) \underbrace{\int \rho(\vec{r}) d^3\vec{r}}_{\equiv Ze} + \frac{e}{6\epsilon_0} \cdot |\psi(0)|^2 \underbrace{\int r^2 \rho(\vec{r}) d^3\vec{r}}_{\equiv Ze \cdot \langle r_c^2 \rangle}$$

Hence, this explain the form of the field shift expression: $\delta\nu_{FS}^{A,A'} = K_{FS} \cdot \delta\langle r_c^2 \rangle^{A,A'}$

The shift is dominant for S state as for P state the electron $|\psi(r)|^2 = 0$ at the origin



Effect of field shift: bring E-level closer to continuum with increasing A

Mass shift

Mass shift comes from the finite nuclear mass

This changes the energy levels as: $E = \frac{-\alpha^2}{2}\frac{m_e}{n^2}\left(1 - \frac{m_e}{m_A}\right)$

The total change in energy:

$$\Delta E = -\frac{p^2}{2m_A} = -\frac{1}{2m_A}(\sum \vec{p_i})^2 = -\frac{1}{2m_A}\sum p_i^2 - \frac{1}{2m_A}\sum \vec{p_i}\cdot\vec{p_j}$$

Normal mass shift → $-\frac{1}{2m_A}\sum p_i^2$

Specific mass shift → $-\frac{1}{2m_A}\sum \vec{p_i}\cdot\vec{p_j}$

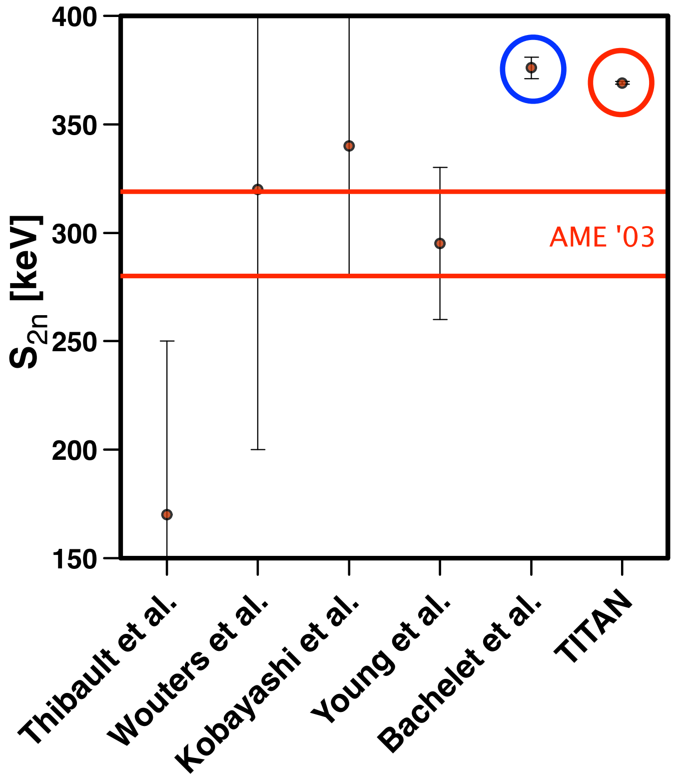
Mass measurement of ^{11}Li

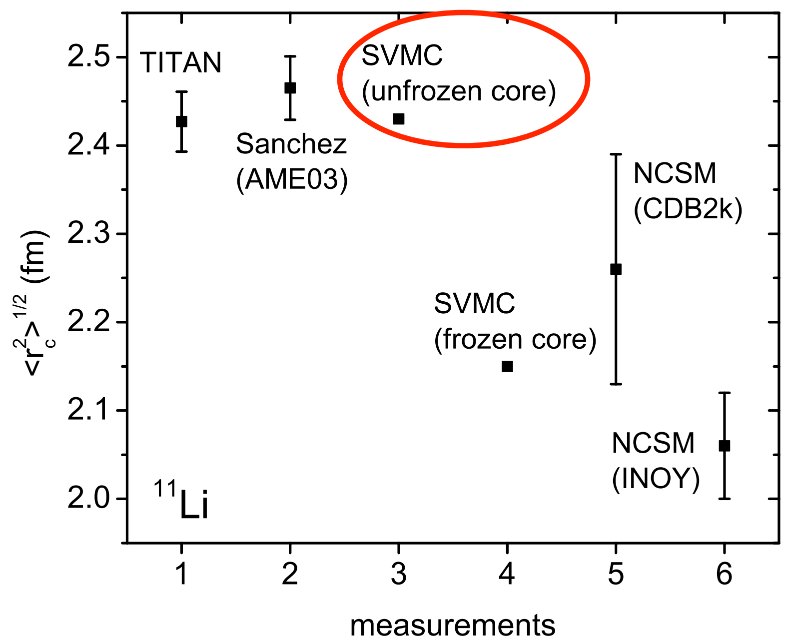
M. Smith et al., Phys. Rev. Lett. 101, 202501 (2008).

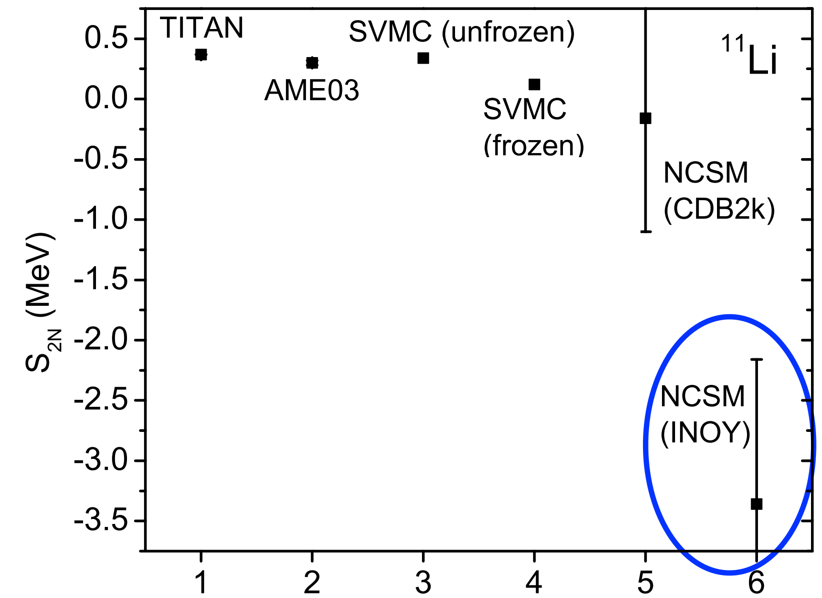
Bachelet et al. measurement shows 65 keV deviation with AME03

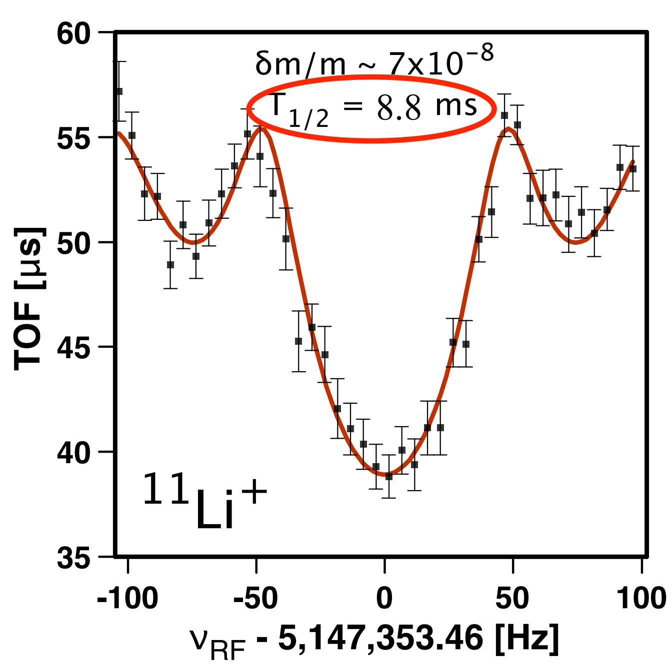
C. Bachelet et al., Phys. Rev. Lett. 100, 182501 (2008)

Confirmed by the TITAN shortest lived mass measurement using Penning trap





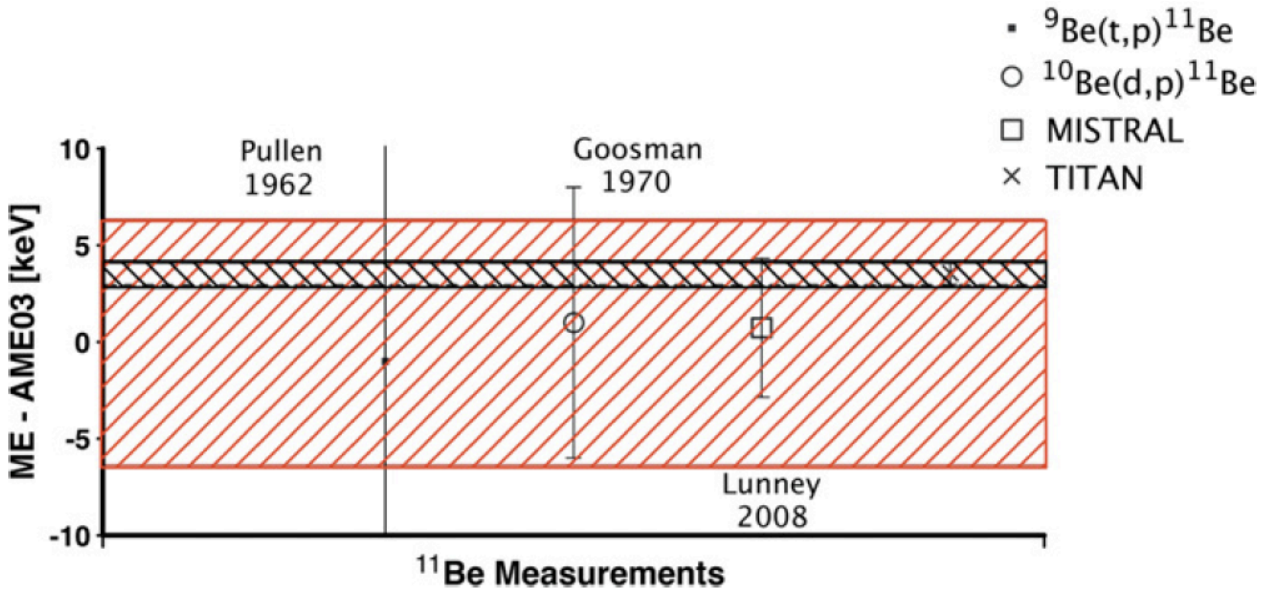


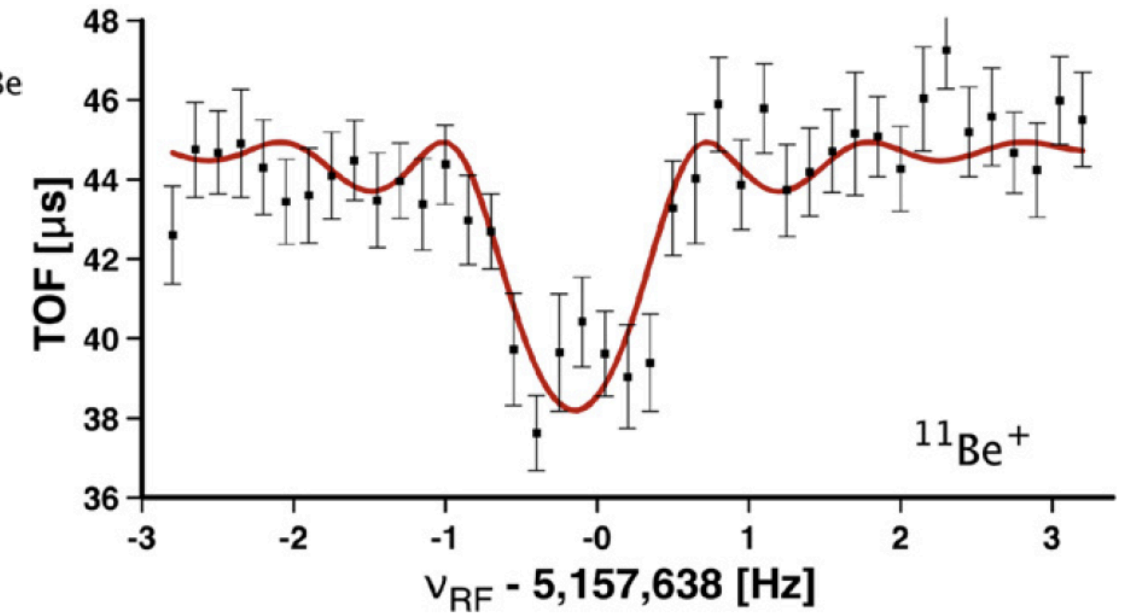


- NCSM (INOY): unbound ^{11}Li with a physical r_c
- Stochastic Variational Monte-Carlo cluster model (SVMC) with unfrozen core gives the best agreement for both r_c and S_{2N}
- The ^{9}Li core should be seen as unfrozen, which means it is allowed to be deformed by the presence of the valence neutrons

Mass measurement of ^{11}Be

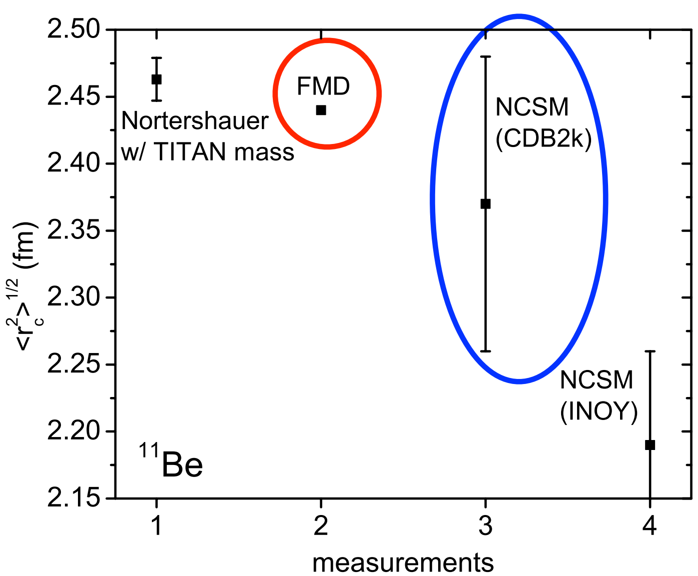
TITAN mass measurement of the one n-halo ^{11}Be

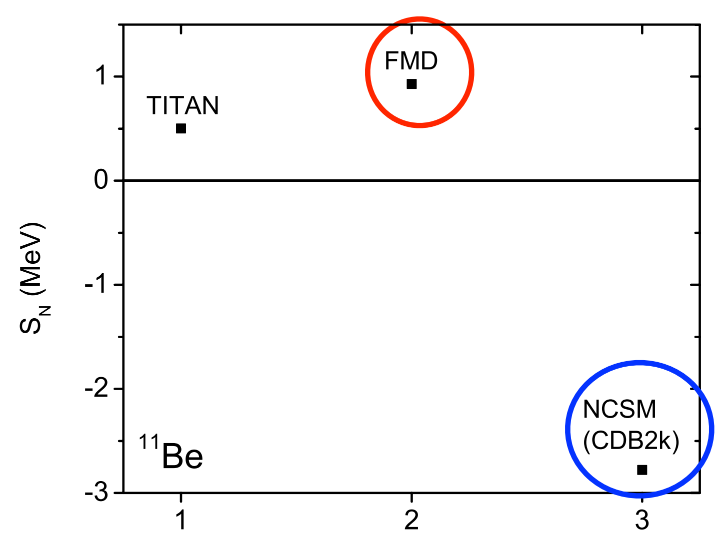




→ Improve precision on the mass by one order of magnitude

→ The latest charge radius determination uses the TITAN mass (Nörtershäuser et. al., PRL **102** (2009) 062503)





NCSM (CDB2k, INOY) r_c ^{11}Be: Forssén *et al.*, PRC **79** (2009) 021303(R)

FMD r_c and S_N ^{11}Be: B.R. Torobi Ph.D. thesis, Darmstadt (2010)

NCSM (CDB2k) S_N ^{11}Be: Quaglioni *et al.*, PRL **101** (2008) 092501

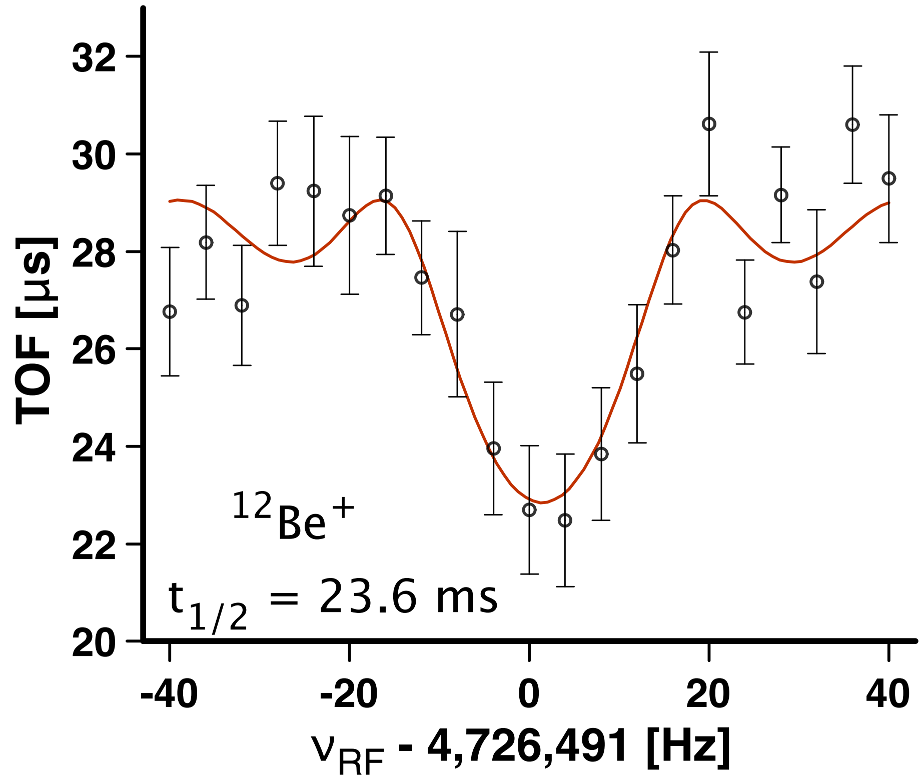
→ NCSM (CDB2k): unbound ^{11}Be with a physical r_c

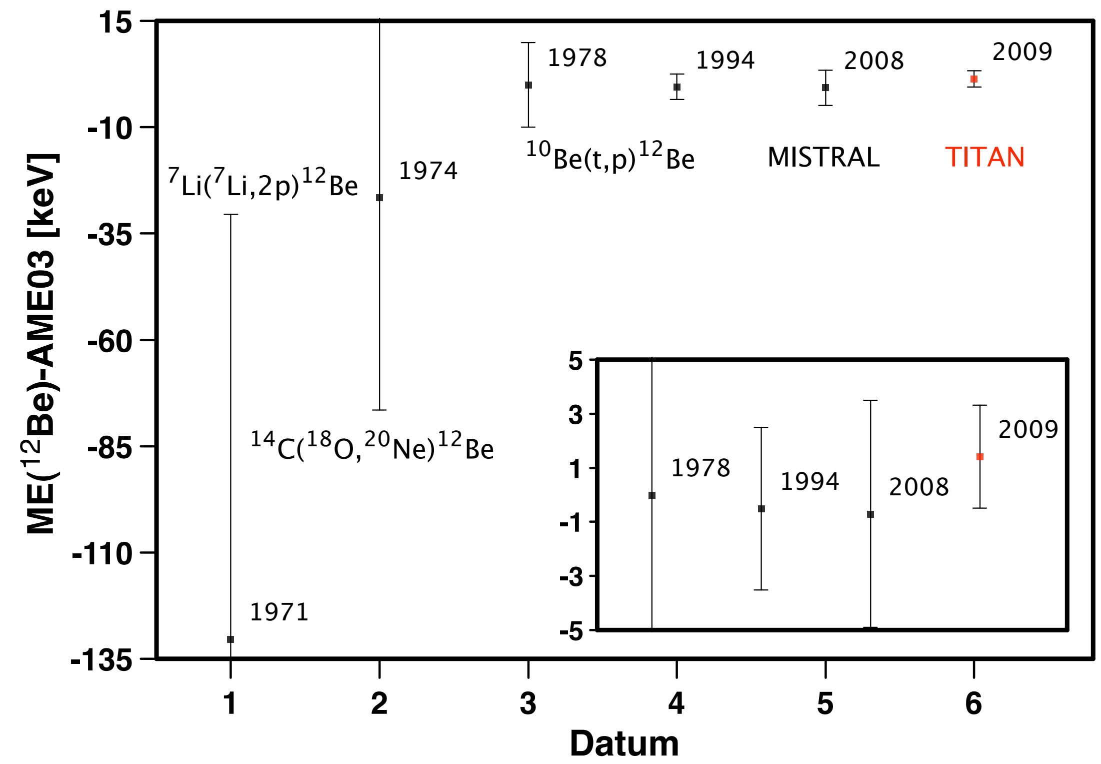
→ Fermionic Molecular Dynamic (FMD) gives the best agreement for r_c and S_N (potential used mimic 3 body interactions)

Mass measurement of ^{12}Be

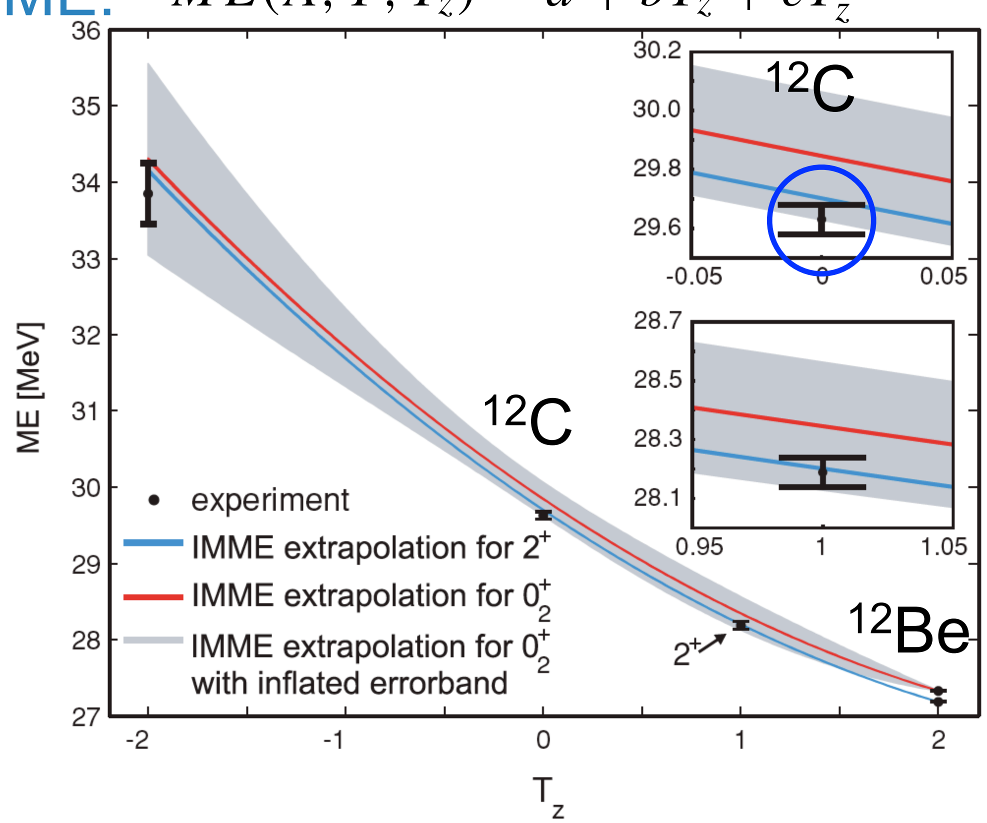
S. Ettenauer et. al, PRC **81** 024314 (2010)

First step towards the mass measurement of the 2n-halo ^{14}Be ($T_{1/2}$ = 4.4 ms)





IMME: $ME(A,T,T_z) = a + bT_z + cT_z^2$



- Dispute regarding the J assignment of ^{12}C (either 0^+ or 2^+)
- Updated the A = 12 (for $J^p = 0^+$) IMME evaluation using the new TITAN ^{12}Be mass
- Using these fit parameters, made prediction that favours the $J^p = 0^+$ state

Ettenauer *et al.*, in prep.

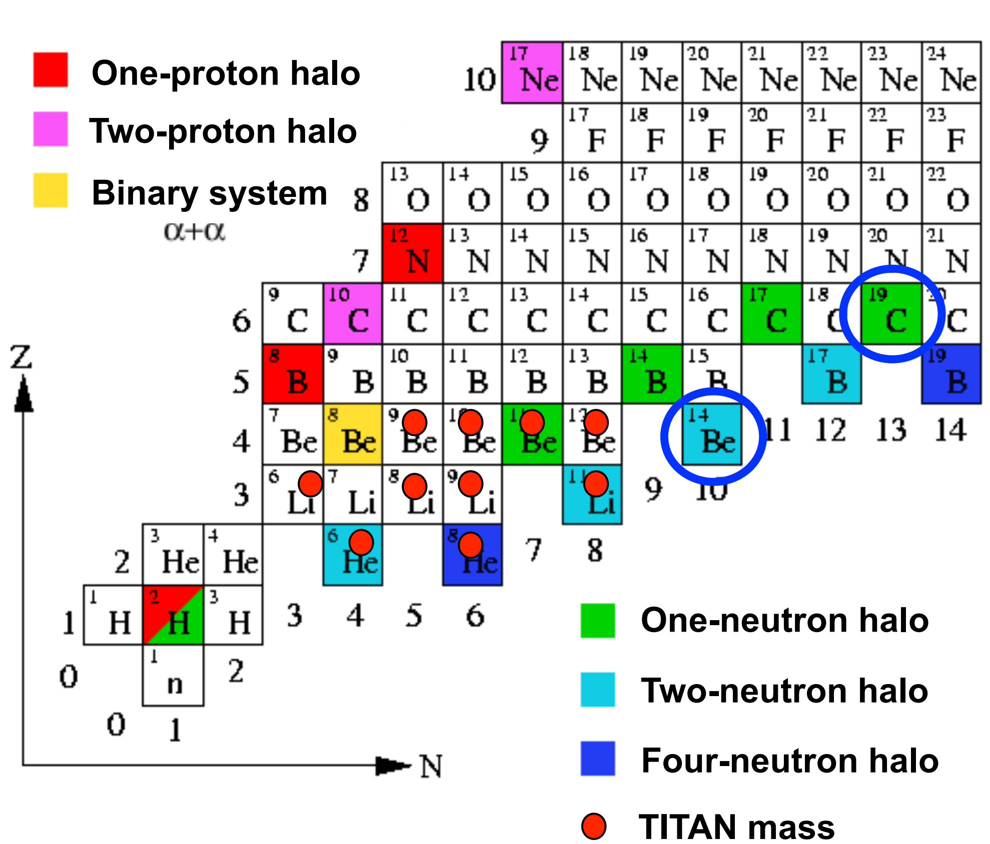
Light masses measured at TITAN

Halo nuclei	Reference	Old AME03 M.E. (keV)	TITAN new M.E. (keV)
He-6	Brodeur et al. in prep. PRL	17595.11 +/- 0.76	17592.087 +/- 0.054
He-8	Ryjkov et al. PRL 08 Brodeur et al. in prep. PRC	31598.0 +/- 6.9	31609.723 +/- 0.106
Li-11	Smith et al. PRL 08	40797 +/- 19	40728.28 +/- 0.64
Be-11	Ringle et al. PLB 09	20174.1 +/- 6.4	20177.60 +/- 0.58

- New level of precision on halo nuclei masses (^{6,8}He, ^{11}Li, ^{11}Be)
- Confirmed the ^{6}Li mass from SMILETRAP, which disagreed from AME03 by 16 ppb → milestone measurement at a precision of 4 ppb
- Improved the precision on the mass of ^{8,9}Li as well as the stable ^{9}Be and nearly stable ^{10}Be
- Measured the mass of 20 ms lived ^{12}Be with count rate of ~ 30 nuclei/s

Halo program plans:

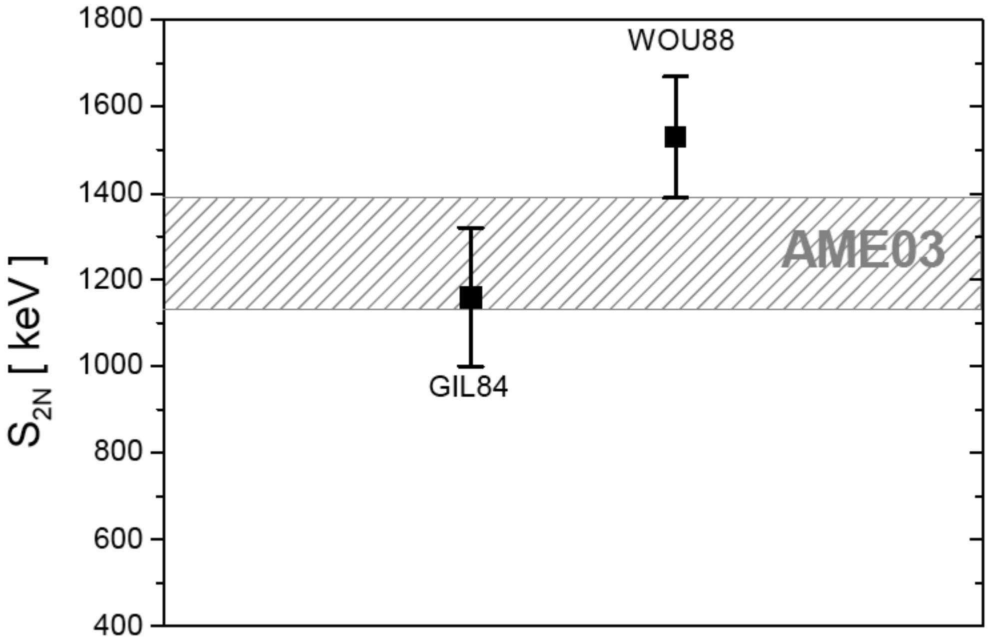
- Proposals to measure the mass of 1n halo ^{19}C and 2n halo ^{14}Be (expect <10 nuclei/s)
- Maybe more out there?



Future measurements at TITAN

Pushing towards ^{14}Be

^{14}Be: 2n-halo nucleus with $T_{1/2}$ = 4.35 ms



- Current mass based on two measurements that differs by 370(210) keV

  [WOU88] J.M. Wouters et al, Z. Phys. A 331 (1988) 229

  [GIL84] R. Gilman et al., Phys. Rev. C 29 (1984) 958

- Cluster model description of this nuclei requires a more reliable mass to constrain their model parameters

  T. Tarutina, I.J. Thompson, J.A. Tostevin, Nucl. Phys. A 733 (2004) 53

Main challenges on production side:

- Thick Ta target hinders release times for short-lived Be isotopes
- Short TaC stack to be used this year

Getting ready on the TITAN side:

- ^{6}Li-^{7}Li frequency ratio test measurement was preformed at 100 Hz
- Expect low yield (10 ions/sec); measured ^{12}Be with 30 ions/sec

Meanwhile, TITAN improved the ^{12}Be mass:

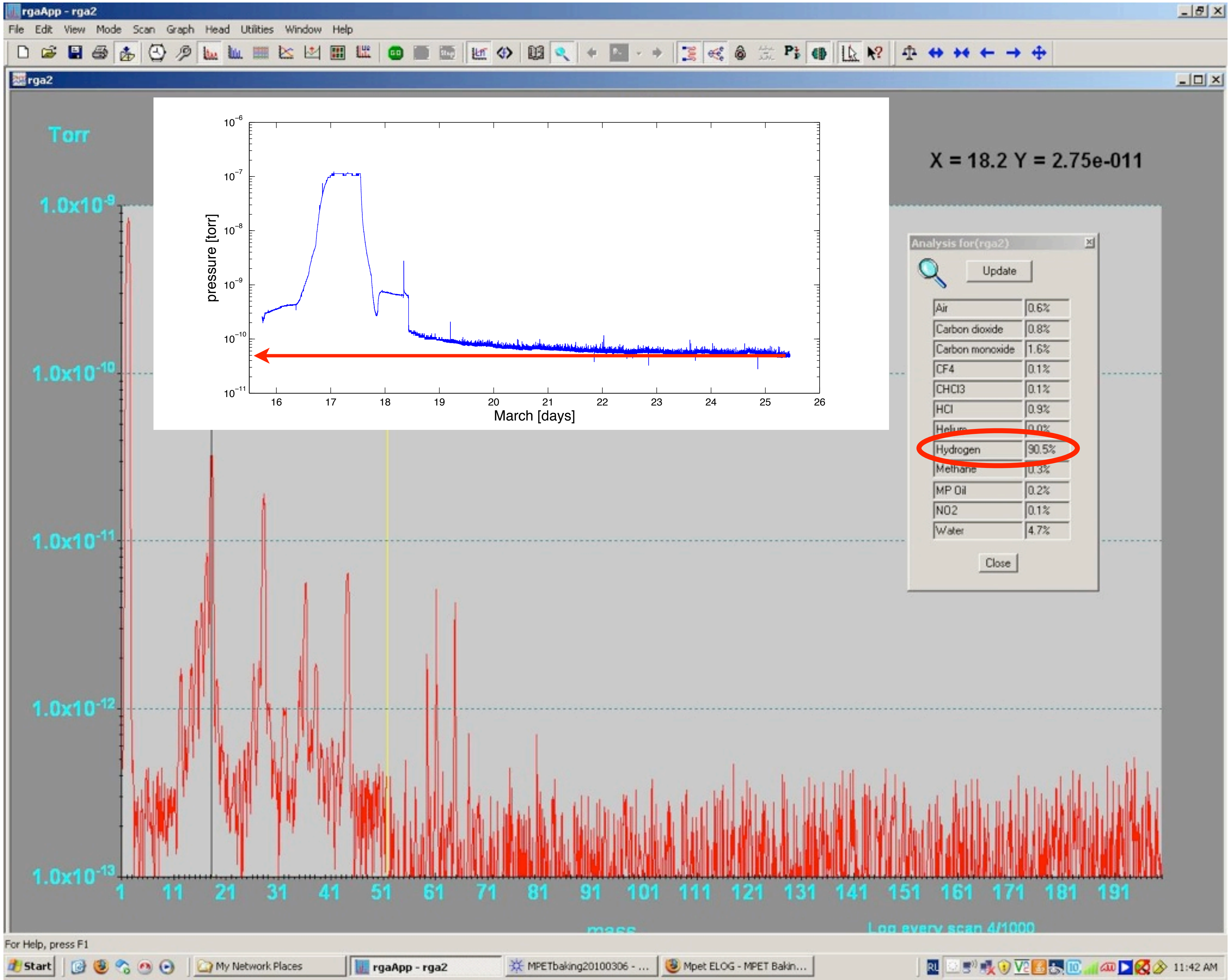
ME(AME03) = 25 076.5(15.0) keV → ME(TITAN) = 25 078.0(2.1) keV

S. Ettenauer et al., PRC 81, 024314 (2010)

Improved ^{12}Be mass value contributes to more precise S_{2n}(^{14}Be)

Baking of the Penning trap

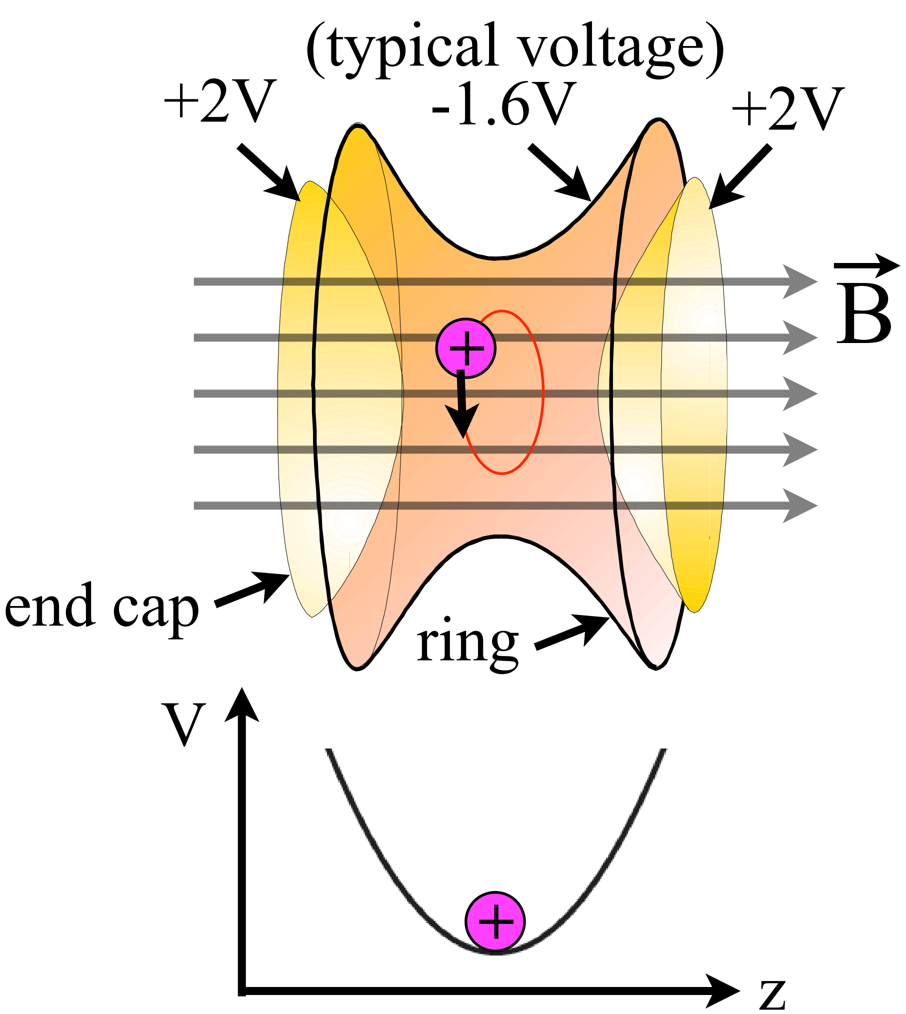
- Baked vacuum tube at 200C (trap centre) for a total of 7 days; mainly H left
- Now pressure reached 4 x 10^{-11} torr



TITAN Penning trap systematic studies

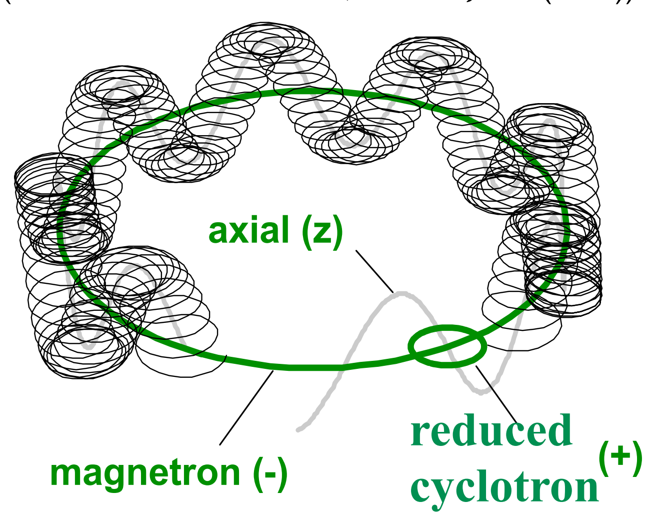
Penning trap basics

Ideal Penning trap:



Ions have 3 eigenmotions:

(L.S. Brown & G. Gabrielse, RMP **58,** 233 (1986))



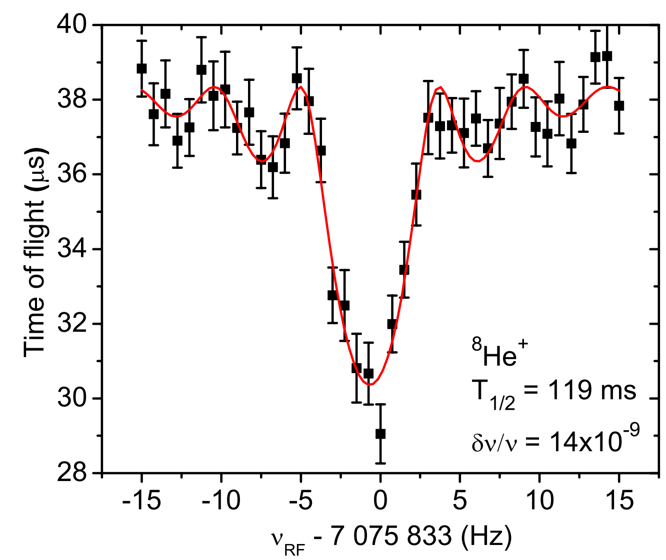
The free-ion cyclotron frequency depends on its mass:

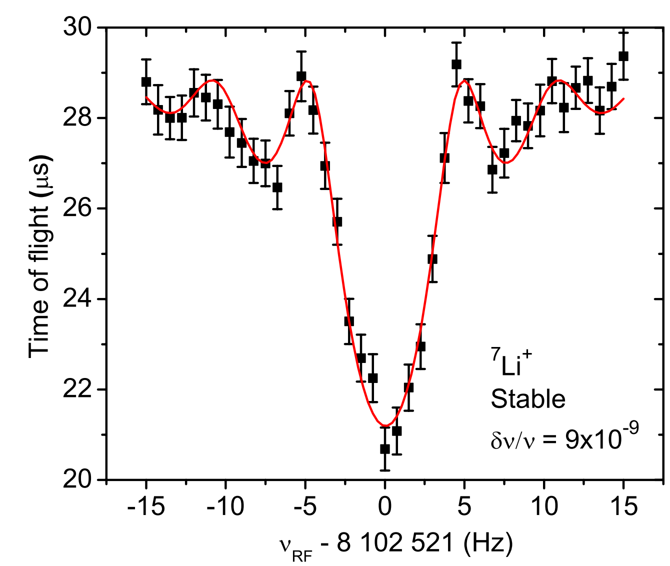
$$\nu_c = \frac{1}{2\pi}\frac{q \cdot B}{M} = \nu_- + \nu_+$$

Ideal trap!

knowing q, B and ν_c one can determine the ion's mass

→ Cyclotron frequency measured using the TOF-ICR technique (G. Graff et al., ZPA **257,** 35 (1980)) (M. Konig et al., IJMSIP **275,** 95 (1995))





→ Magnetic field determined from calibrant ion cyclotron frequency

$$R = \nu_{c,inter}/\nu_c$$

→ Finally, evaluate the atomic mass:

$$m = \overline{R} \cdot (m_{cal} - m_e + B_{e,cal}) + m_e - B_e$$

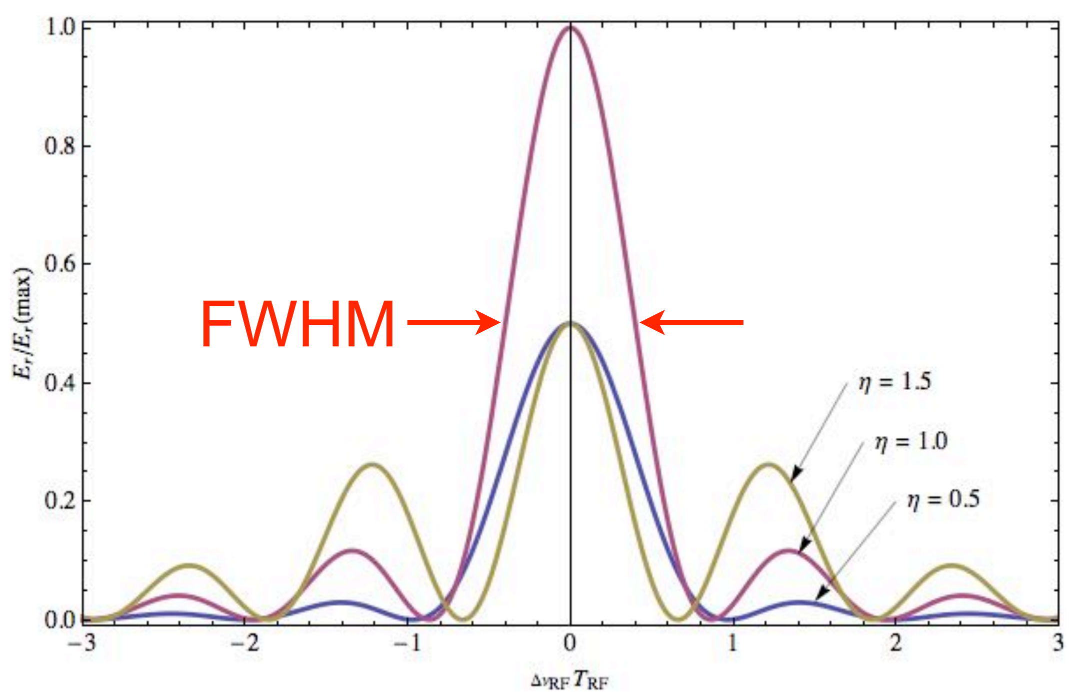
E_r vs detuning frequency

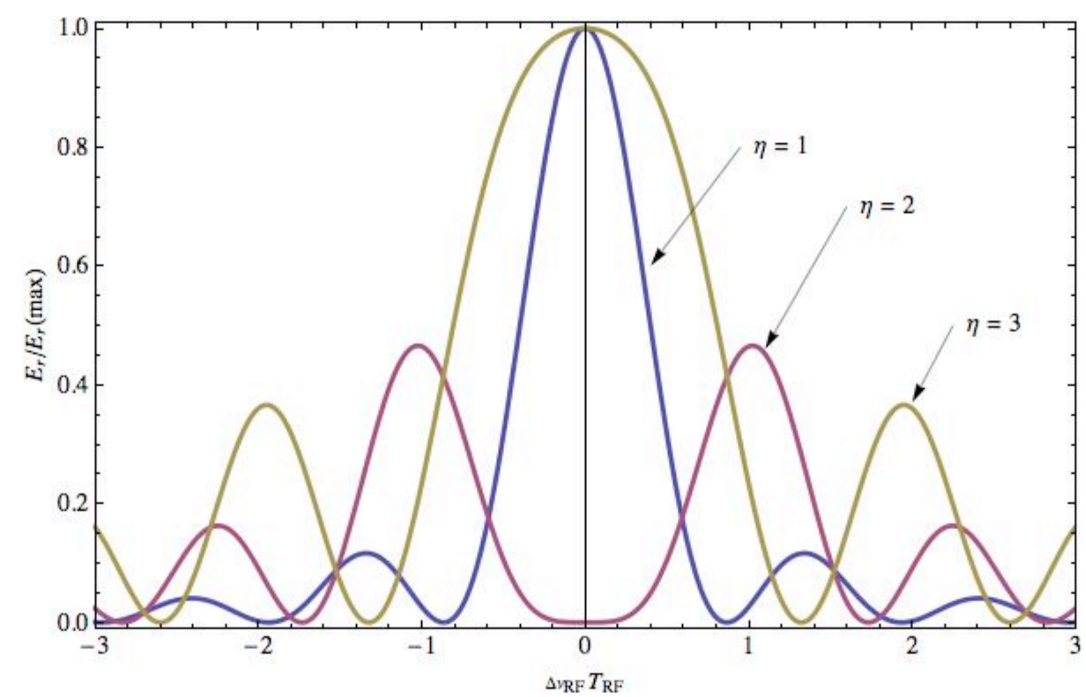
The ion's kinetic energy in the radial plane as function of the detuning $\Delta\nu_{RF} = \dfrac{\omega_{RF} - \omega_c}{2\pi}$

Sinc profile coming from the constant RF amplitude applied over a finite time.

$$E_r = E_0 \frac{\sin^2\left(\frac{\pi}{2}\sqrt{(2\Delta\nu_{RF}T_{RF})^2 + \eta^2}\right)}{(2\Delta\nu_{RF}T_{RF}/\eta)^2 + 1}$$

conversion factor





full conversion for: $\eta = k_0 T_{RF}/\pi = 1, 3, 5\ldots$ but FWHM minimal for $\eta = 1$

in which case, $\Delta\nu \cdot T_{RF} \approx 0.8$

statistical uncertainty of mass measurement: $\dfrac{\delta m}{m} \approx \dfrac{1}{\mathscr{R}}\dfrac{1}{\sqrt{N_{ion}}}$

where, the resolving power R: $\dfrac{1}{\mathscr{R}} = \dfrac{\Delta\nu}{\nu_c} = \dfrac{0.8}{T_{RF}} \cdot \dfrac{2\pi m}{qB}$ ⇒ $\dfrac{\delta m}{m} \approx \dfrac{1.6 \cdot \pi \cdot m}{q \cdot B \cdot T_{RF} \cdot \sqrt{N_{ion}}}$

Accuracy of the system

Depending on the count rate and excitation time, TITAN Penning trap can achieve precision in the ppb range for A < 10.

$$\frac{\delta m}{m} \approx \frac{m}{q \cdot B \cdot T_{RF} \cdot \sqrt{N_{ion}}}$$

→ But need to determine if the system is also accurate at this level!

→ To do so, several sources of systematic errors were investigated, including: (for the 3.6 V trap depth used for the halo mass measurements)

Recall: $R = \nu_{c,inter}/\nu_c$ $\quad m = \overline{R} \cdot (m_{cal} - m_e + B_{e,cal}) + m_e - B_e$

Error	$\Delta R/R$ ($\times 10^{-10}$)
magnetic field inhomogeneities	$0.2 \cdot \Delta A$
misalignment and harmonic distortion	$4.2 \cdot \Delta A$
incomplete compensation	$0.5(5) \cdot \Delta A$
non-linear magnetic field fluctuation	$1.5 \cdot \Delta t$ (h)

(M. Brodeur Ph.D. thesis, UBC (2010))

As well as other sources of errors that can be minimized during the measurement:

→ Relativistic effects (adjusting ion radius such they have similar velocity)

→ Ion-ion interaction (adjust count rate such as to have mainly one ion at the time)

Misalignment & harmonic distortion

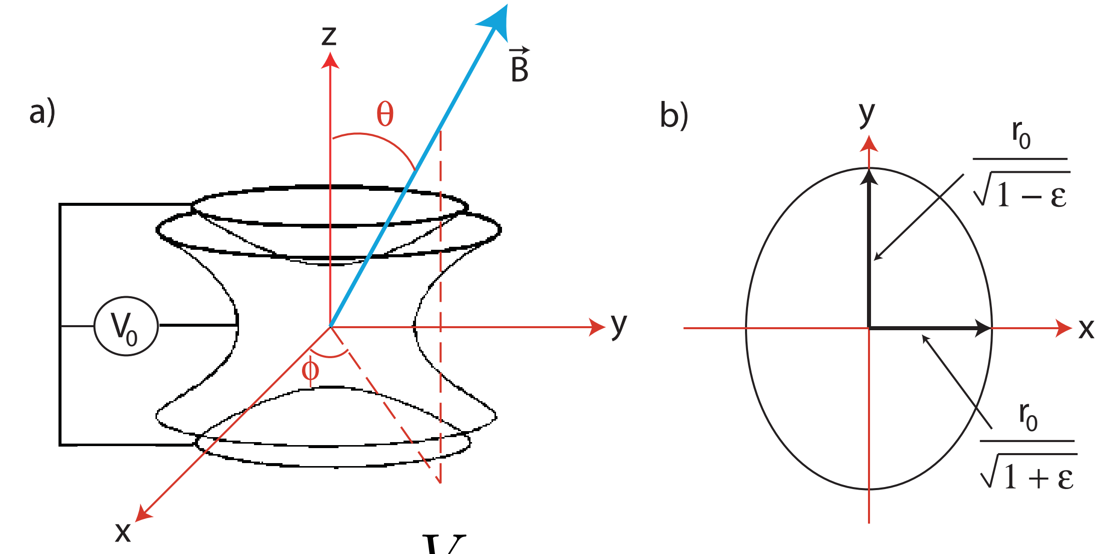
Trap misalignment with the B-field (a) and harmonic distortion of the trapping potential (b) change the eigen frequencies such that: $\bar{\nu}_- + \bar{\nu}_+ = \bar{\nu}_c \neq \frac{1}{2\pi}\frac{q \cdot B}{M}$

→ Using the Invariance Theorem
(L.S. Brown & G. Gabrielse, PRA **25,** 2423 (1982))

$$\nu_c^2 = \bar{\nu}_-^2 + \bar{\nu}_+^2 + \bar{\nu}_z^2$$

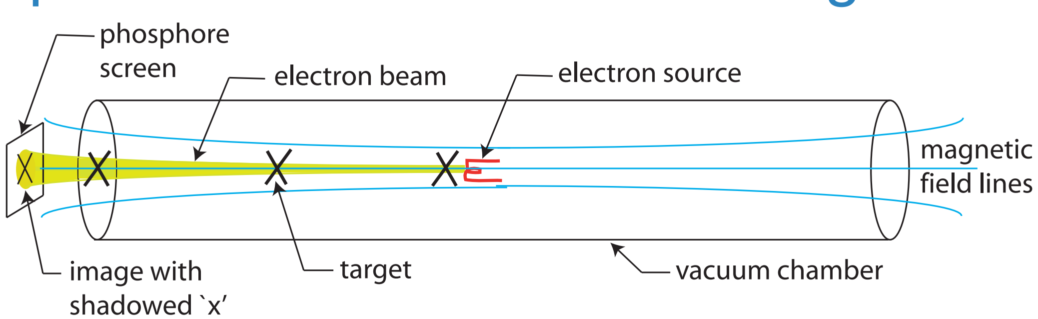
The corresponding systematic error on the frequency ratio *R* is given by:
(G. Gabrielse, PRL **102,** 172501 (2009))

$$(\Delta R/R)_{mis.} = \left(\frac{9}{4}\theta^2 - \frac{1}{2}\epsilon^2\right) \cdot \left(\frac{\Delta A}{A_{cal.}}\right) \cdot \left(\frac{\bar{\nu}_-}{\bar{\nu}_{+,cal}}\right)$$



$$V_{harm.dist.} = \frac{V_0}{4d_0^2}\{(1+\epsilon)x^2 + (1-\epsilon)y^2\}$$

→ Minimized the misalignment θ by a precise vacuum chamber alignment



→ and tight machining tolerances

leading (estimated): $\theta_{max} = 4 \times 10^{-3}$

→ Minimize harmonic distortion ε by having a one-piece ring electrode

→ gold-plating to reduce surface imperfections

→ having tight machining tolerances

leading (measured): $\epsilon = 3(2) \times 10^{-4}$

overall: $(\Delta R/R)_{mis.} = 4.2 \times 10^{-9} \cdot \Delta A$ $V_0 = 36V$

$0.4 \times 10^{-9} \cdot \Delta A$ $V_0 = 3.6V$

Non-harmonic imperfections

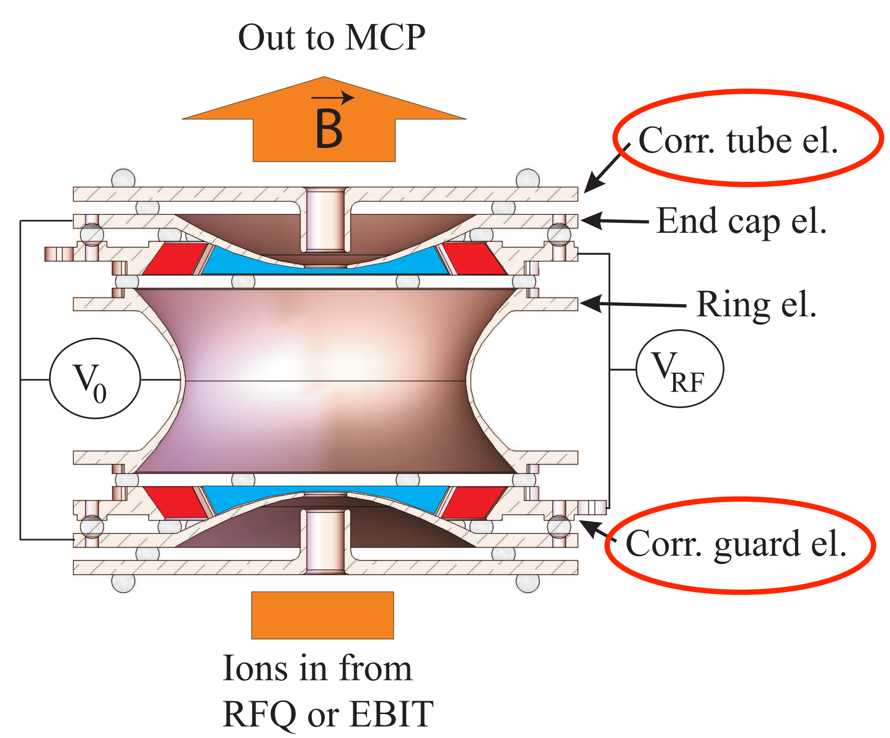
Trapping potential of a real PT is not purely harmonic: $V(z) = \frac{V_0}{2}\left(C_0 + \frac{C_2}{d^2}z^2 + \frac{C_4}{d^4}z^4 + \frac{C_6}{d^6}z^6 + \ldots\right)$

due to:

- → Hole in end caps (ion insertions)
- → Truncation of hyperbola

correct with:

- → tube electrode before end caps
- → guard electrode between end cap and ring



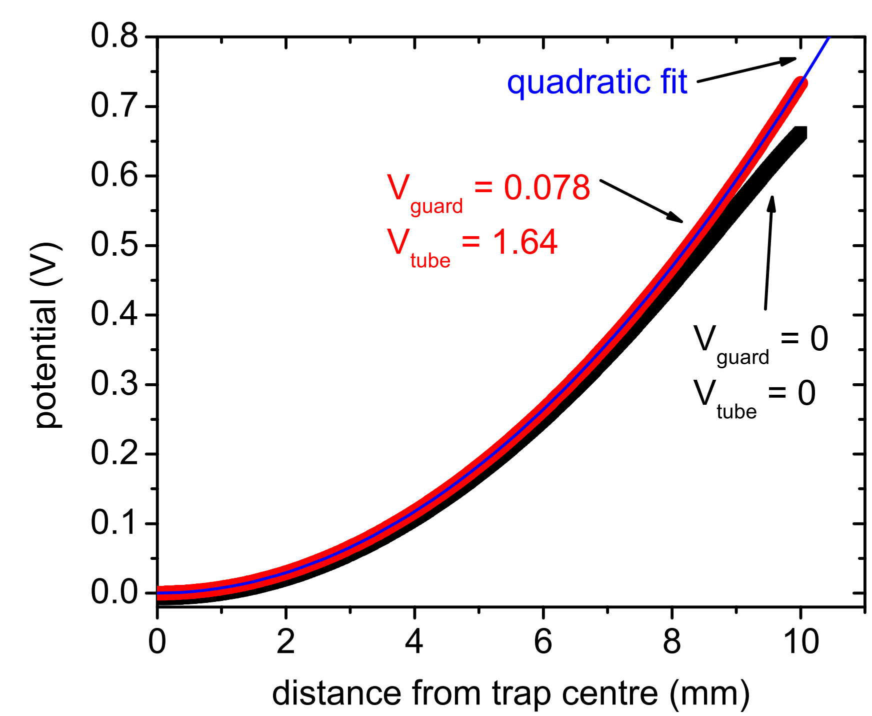
This induce nearly mass-independent frequency shifts: (L.S. Brown & G. Gabrielse, RMP **58,** 233 (1986)) (G. Bollen et al., JAP **68,** 4355 (1990))

$$\delta\nu_c \approx \frac{3}{4}\frac{(r_-^2 - r_+^2)}{d^2}\nu_-\{C_4 + \frac{5}{2}\frac{C_6}{d^2}(3z^2 - r_+^2 - r_-^2) + \ldots\}$$

Resulting in a frequency ratio change: $\frac{\Delta R}{R} = \frac{\Delta\nu_c}{\nu_c \cdot A} \cdot \Delta A$

Possible shifts: $\Delta R/R > 10^{-7}$

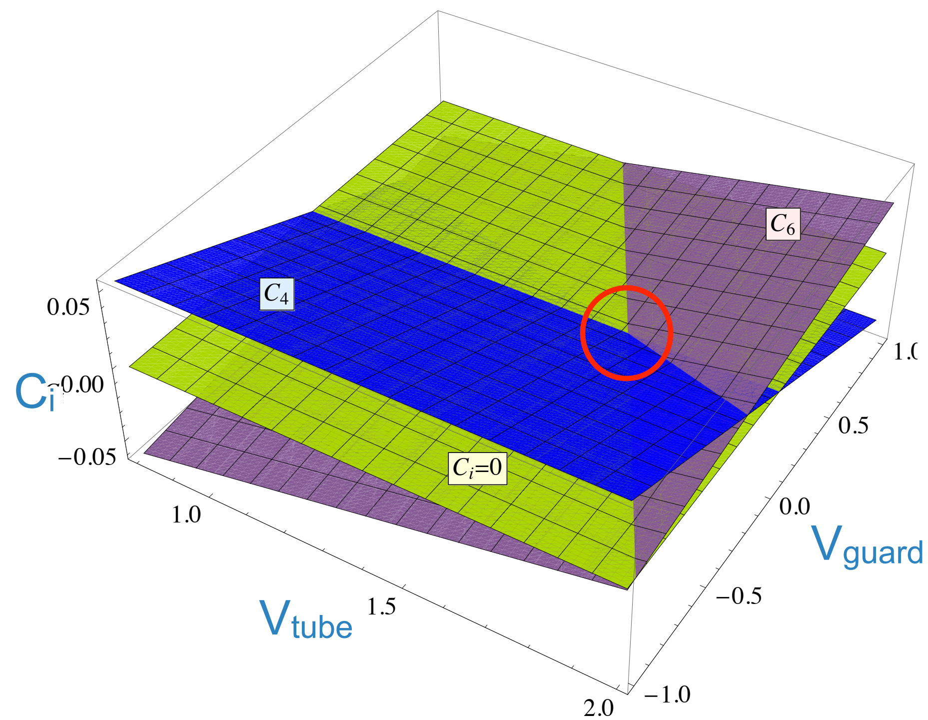
- → Important to compensate the trap potential!
- → Done by adjusting the correction electrodes potential (V_{guard}, V_{tube})



Penning trap compensation

Goal: minimize non-harmonic coefficient C_i by changing V_{tube}, V_{guard}

Dominant terms are C_4 & C_6: $V(z) \simeq \frac{V_0}{2}\left(C_0 + \frac{C_2}{d^2}z^2 + \frac{C_4}{d^4}z^4 + \frac{C_6}{d^6}z^6\right)$



→ C_4, C_6 coefficients depends linearly on V_{tube}, V_{guard}

→ Only one (V_{tube}, V_{guard}) potential sets leads to an optimal compensation of both C_4 & C_6.

→ Thus, needs 2 optimization methods to get the correct compensation

Compensation procedure I

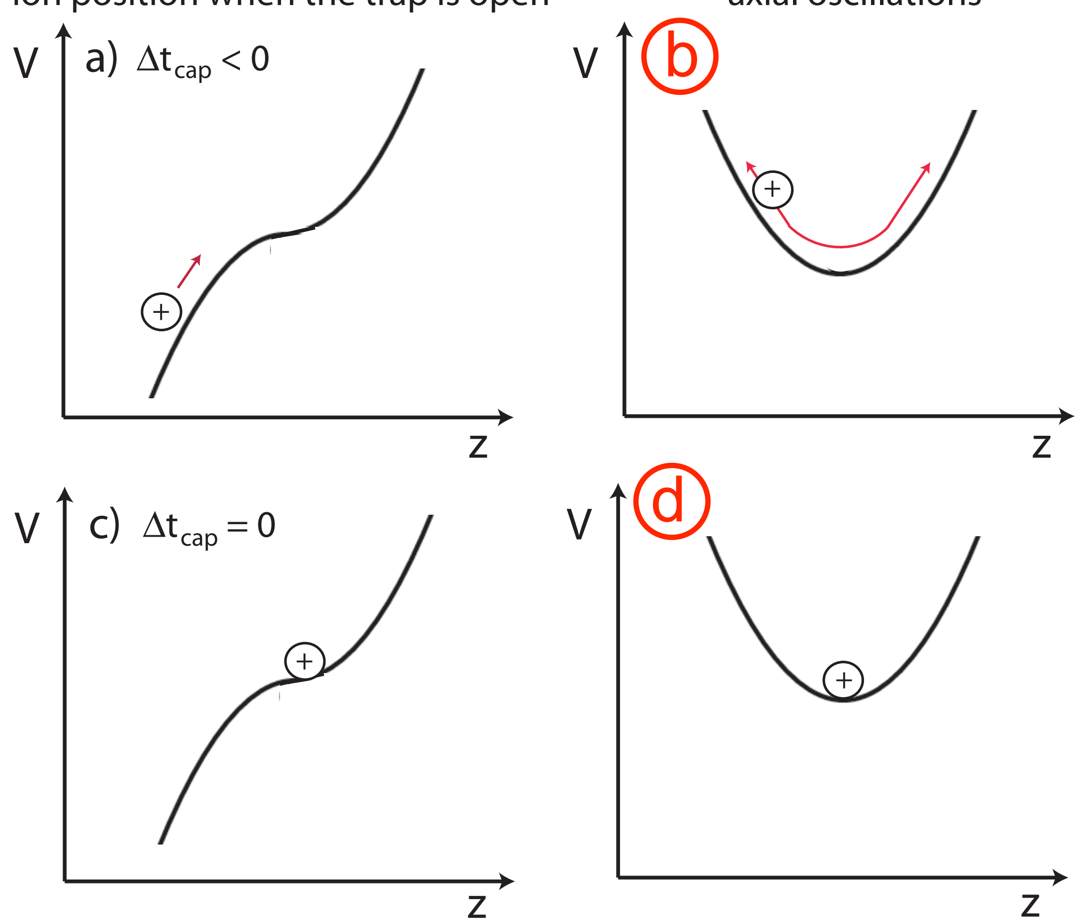
1) Compensation using a dipole excitation (D. Beck et al., NIMA **598,** 635 (2009))

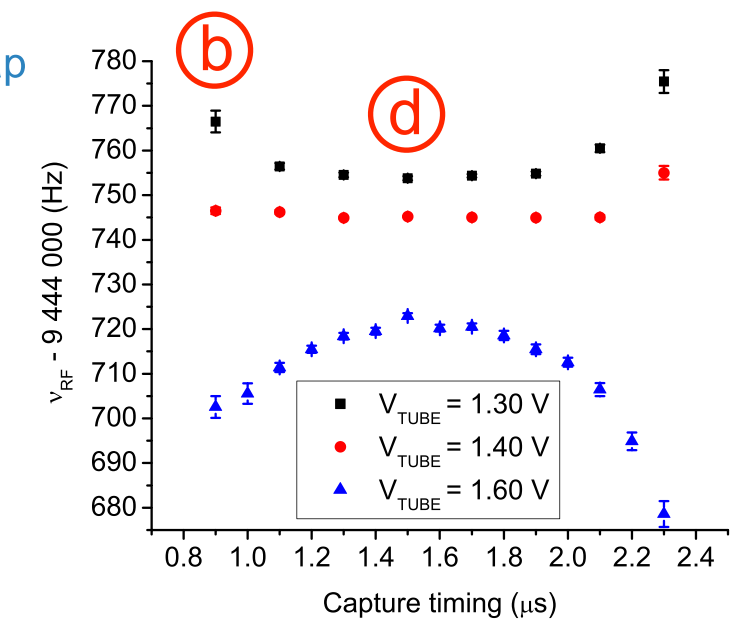
why ν_+ ? → Sensitive on potential: $\nu_+ \approx \nu_c - V_0/(4\pi B d^2)$

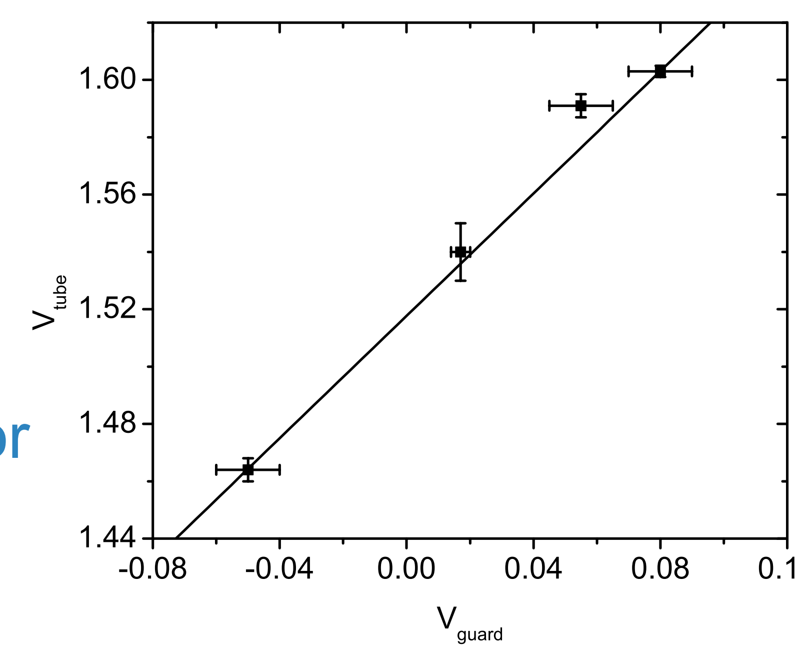
Procedure: find (V_{tube}, V_{guard}) that minimizes change in ν_+ with z

$$\delta\nu_+ \approx \frac{3}{4}\frac{C_4}{d^2}\nu_- \left\{(r_+^2 + 2r_-^2) - 2z^2\right\} + \ldots$$

Done by comparing ν_+ for different trap closing time t_{cap}







→ As expected, observe several optimal settings for (V_{tube}, V_{guard}) that follows a straight line

→ Which one is the correct setting?

Compensation procedure II

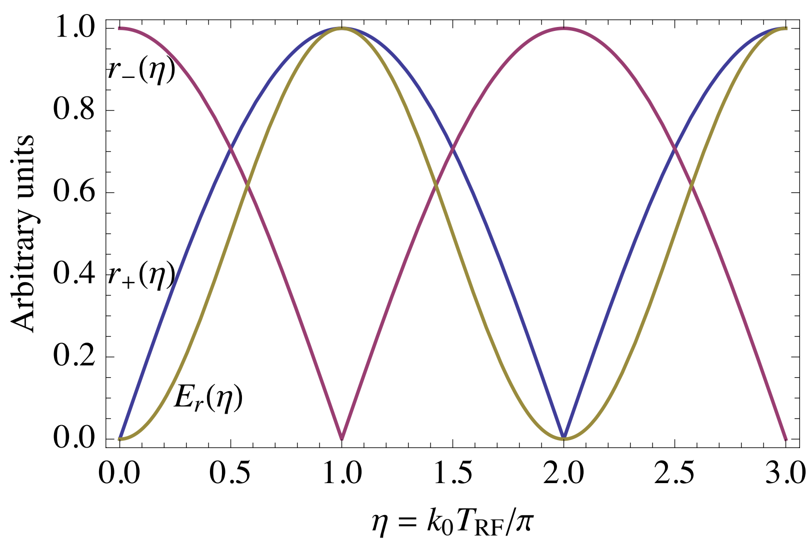
To find the correct setting, need a second compensation procedure.

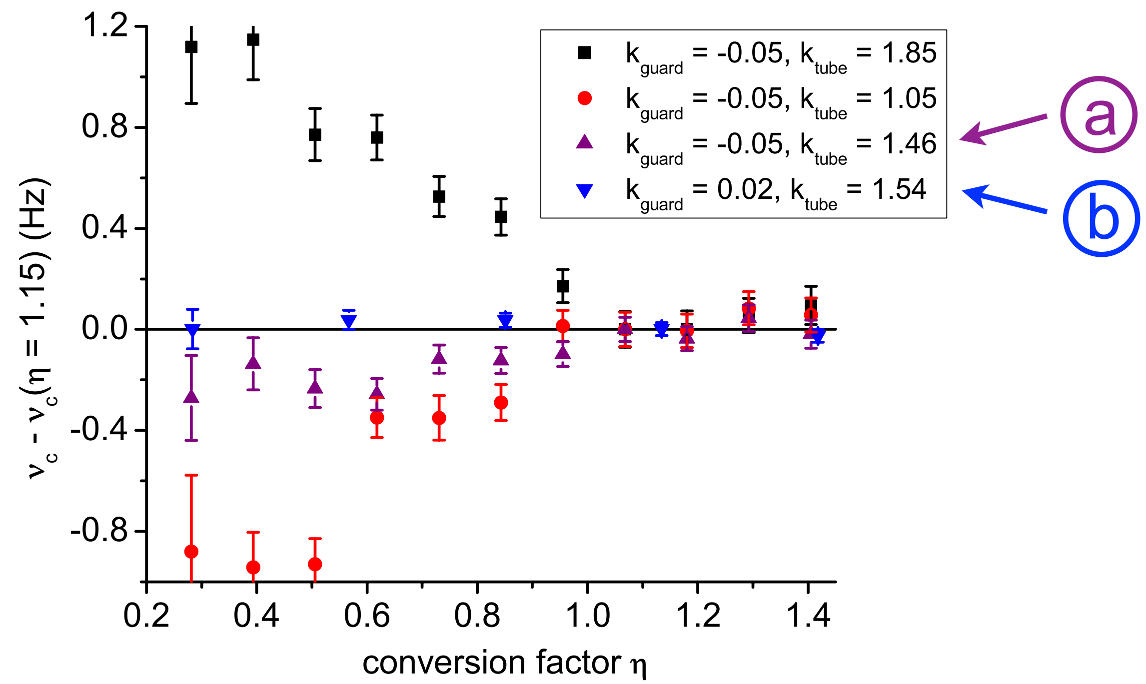
2) Compensation using a quadrupole excitation

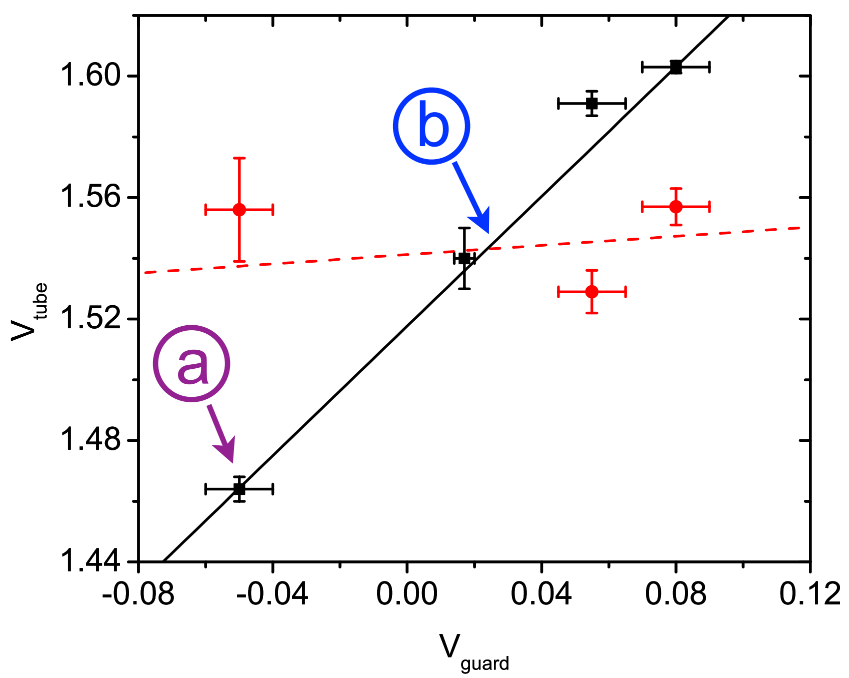
Procedure: find (V_{tube}, V_{guard}) that minimizes change in ν_c with $(r_-^2 - r_+^2)$

$$\delta\nu_c \approx \frac{3}{4}\frac{(r_-^2 - r_+^2)}{d^2}\nu_-\{C_4 + \frac{5}{2}\frac{C_6}{d^2}(3z^2 - r_+^2 - r_-^2) + ...\}$$

Done by comparing ν_+ for different conversion factor η (or RF amplitude)







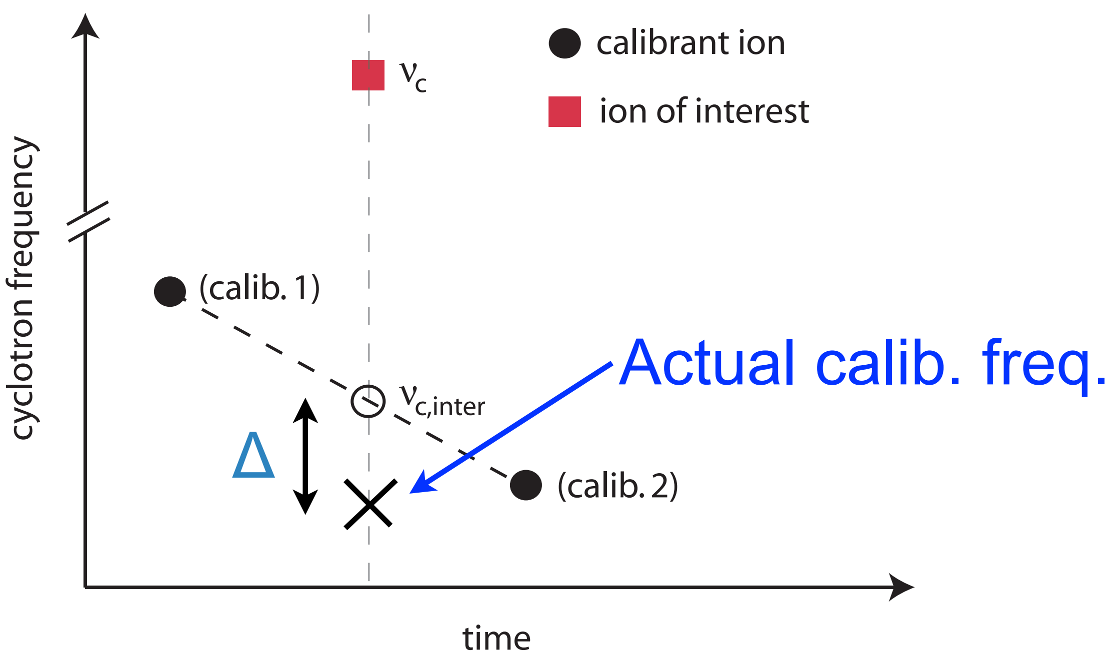
→ Cannot get the optimal compensation by only using one compensation procedure.

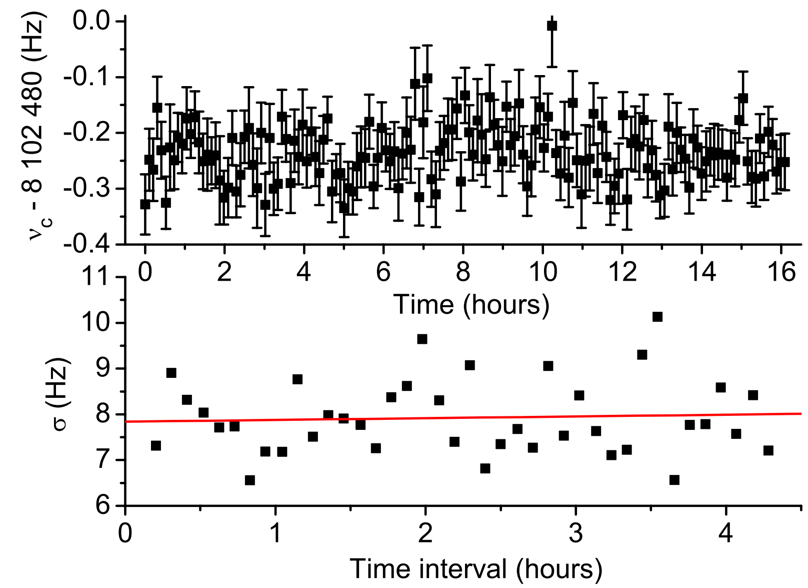
→ results from frequency ratio measurements of ^{39}K vs ^{23}Na and ^{23}Na vs H_3O in a systematic error of:

$$\frac{\Delta\nu_c}{\nu_c \cdot A} = -0.5(5)\ \mathrm{ppb/u}$$

Magnetic field fluctuations

Recall the frequency ratio measurement procedure:





Calculate the error due to the interpolation procedure:

1) Measure ν_c for a long period of time (larger than typical meas. period)

2) Interpolate ν_c between alternative measurements

3) Calculate the difference Δ between interpolation and real measurement

4) Calculate the the spread σ of the corresponding gaussian distribution

5) Repeat for larger separation between calibrations

6) Plot change in spread over time, slope would give error on interpolation.

For TITAN, by measuring the ν_c of ^{7}Li for 16h, found: $\delta\nu/\nu = 0.04(11)$ ppb/h

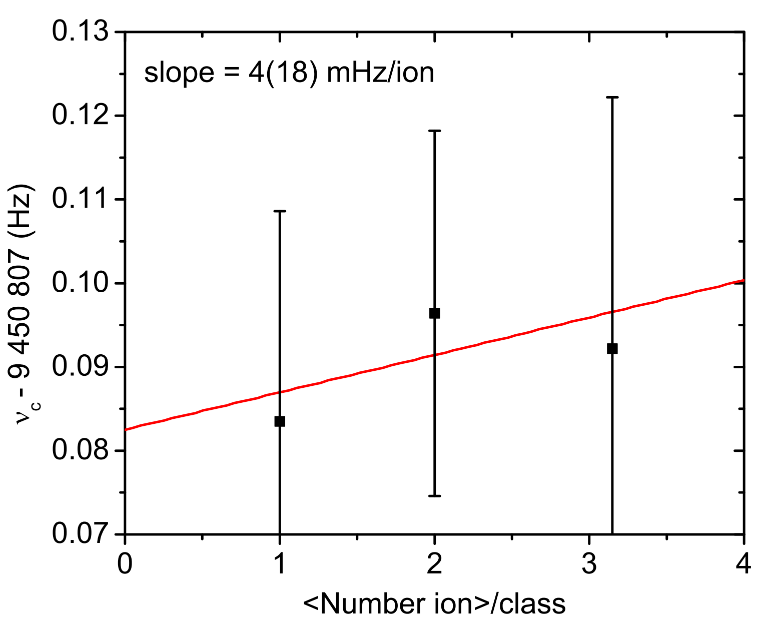
Ion-ion interaction

When two ions of different masses are present in the trap, the unexcited ion will be seen as a positively charged ring.

This charged ring modifies the potential seen by the ions (no longer harmonic), which will change the measured cyclotron frequency.

This shift of the cyclotron frequency is determined by breaking the TOF spectra according to the number of detected ions (1,2,3,...) and fitting these spectra.

Typical example for ^{6}Li:



To find the error, we perform such fit for several TOF spectra and by taking the weighted mean of the obtained slopes. Results:

Specie	Slope $\Delta\nu_c$ (mHz/ion)	N
^{7}Li	-1.4(2.9)	115
^{6}Li	8.4(3.2)	77
Both Li	3.1(2.2)	189
^{4}He	-18.5(14.2)	5
^{6}He	-9.3(51.4)	8
^{8}He	100.2(150.0)	5

Note these are conservatives upper values

Then, calculate the error on R:

$$(\Delta R/R)_{ion} = (N_{cal.} - \varepsilon)\Delta\nu_{c,cal.}/\nu_{c,cal.} - (N - \varepsilon)\Delta\nu_c/\nu_c$$

MCP efficiency: ~60%

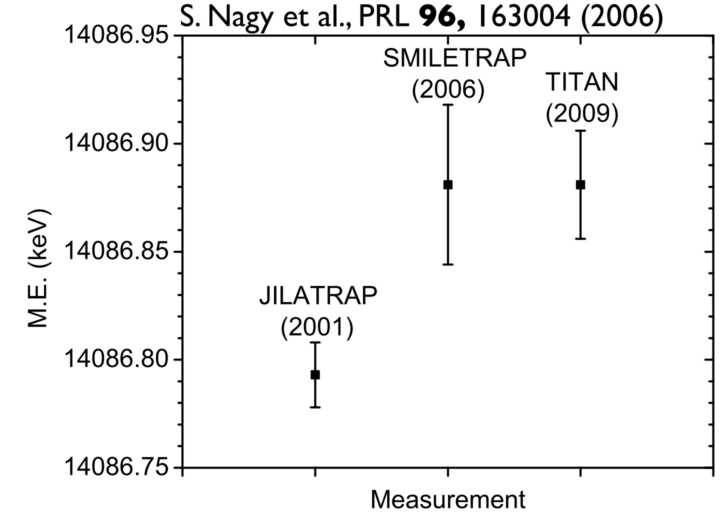
Specie	N	$(\Delta R/R)_{ion}$ (ppb)
^{4}He	2	4.3
^{6}Li	3	0.2
^{6}He	2	8.1
^{8}He	1	13.3

Mass measurement of ^{6}Li

(M. Brodeur et al., PRC **80,** 044318 (2009))

Two previous ^{6}Li masses disagree by 16 ppb:

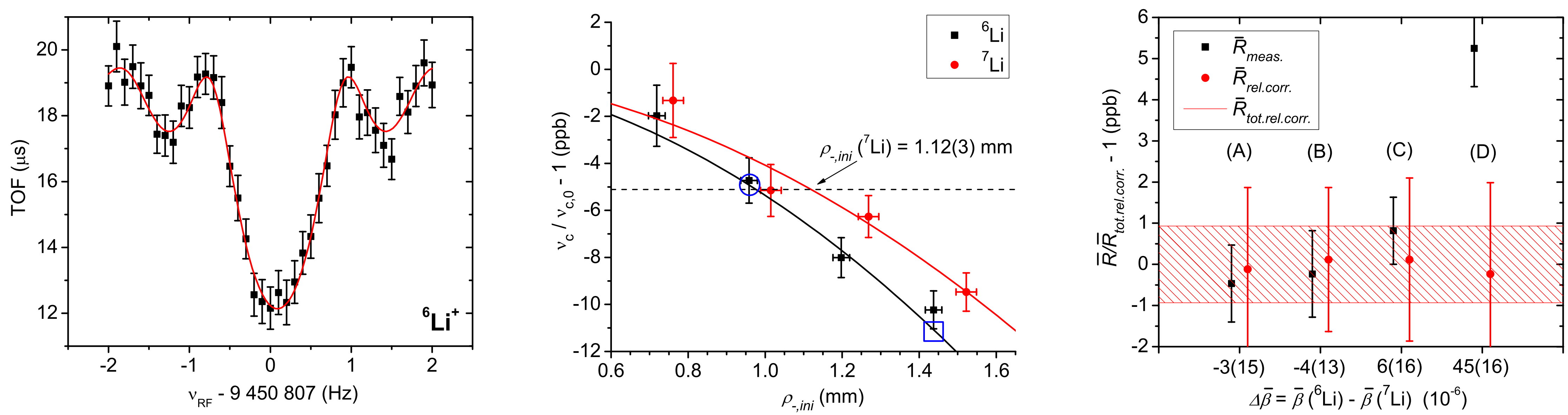
T.P. Heavner et al., PRA **64,** 062504 (2001)
S. Nagy et al., PRL **96,** 163004 (2006)



TITAN performed a mass measurement of ^{6}Li using ^{7}Li as calibrant (both ions produced by off-line ion source)

Light ions like ^{6}Li and ^{7}Li are affected by relativistic effects, which changes the frequency ratio: $R_{rel.} = R_{non.rel.}\sqrt{\frac{1-\overline{\beta}_{cal}^2}{1-\overline{\beta}^2}}$

Solution: adjust $r_{-,ini}$ of ^{6}Li and ^{7}Li in order to have $\overline{\beta}_{cal} \approx \overline{\beta} = r_{-,ini}\cdot 2\pi\cdot\nu_+/c$



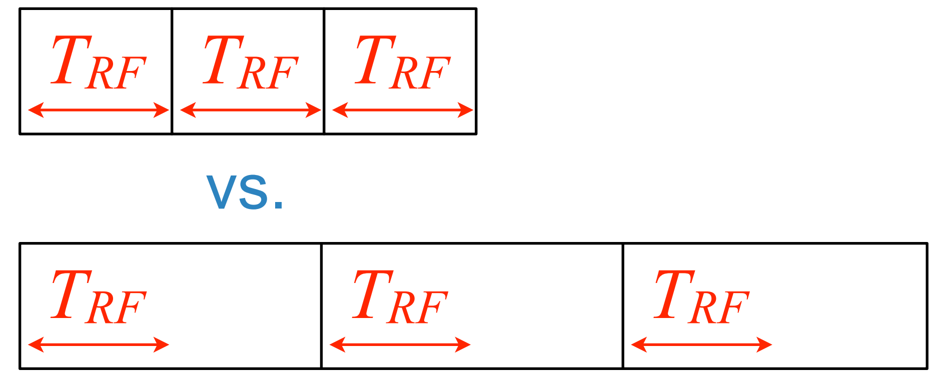
The resulting ^{6}Li TITAN mass confirmed the SMILETRAP value

Fast measurements at TITAN

What makes precise mass measurement on $T_{1/2}$ < 50 ms isotopes possible at TITAN

1) Fast data acquisition and controls

- → Does not limit the measurement repetition rate (can reach 100 Hz)
- → Maximized the measurement time/dead time ratio

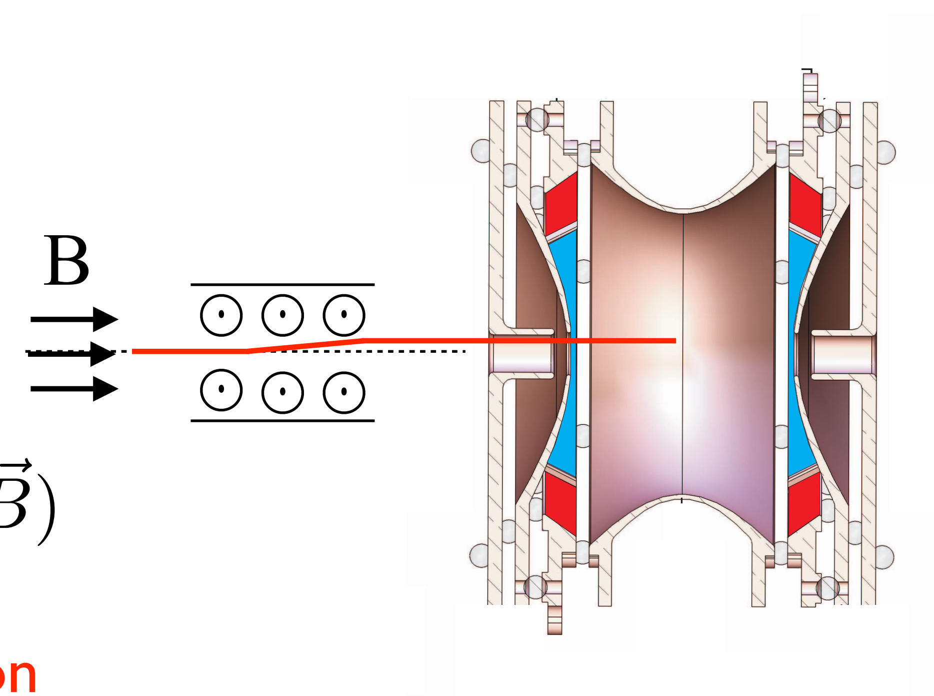


2) Parallel operation

- → Parallel loading of the RFQ
- → Parallel charge breeding in EBIT (if needed)

3) Fast magnetron motion preparation

- → In-flight preparation using a Lorentz steerer R. Ringle *et al.*, IJMS **263** (2007) 38
- → Save on in-trap preparation



$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

→ net offset in **ExB** direction

Comparison with other ab-initio methods

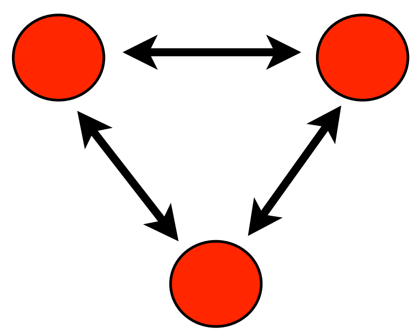
Ab-initio methods

Nuclear theories we test: ab-initio methods (from first principle)

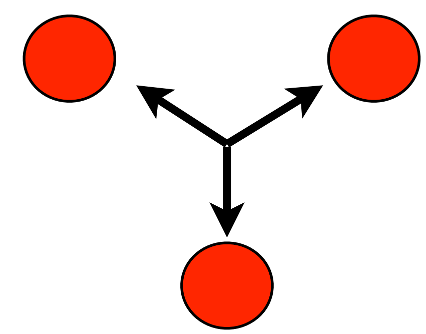
- Treat all the nucleons on the same footing
- Calculate properties by solving S.E. $H\Psi = E\Psi$
- Need a potential and to construct the wave function

$$H = T + V = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V_{ij}^{2N} + \sum_{i<j<k} V_{ijk}^{3N} + \ldots$$

2-body interactions



3-body interactions



Ab-initio methods for ^{6,8}He:

GFMC: Green Function Monte Carlo method, uses V2N (AV18) and V3N (ILs).

NCSM: No-core shell model method, uses V2N only (CD-Bonn 2000 or INOY).
note: wave function present Gaussian fall-out (halo: exponential)

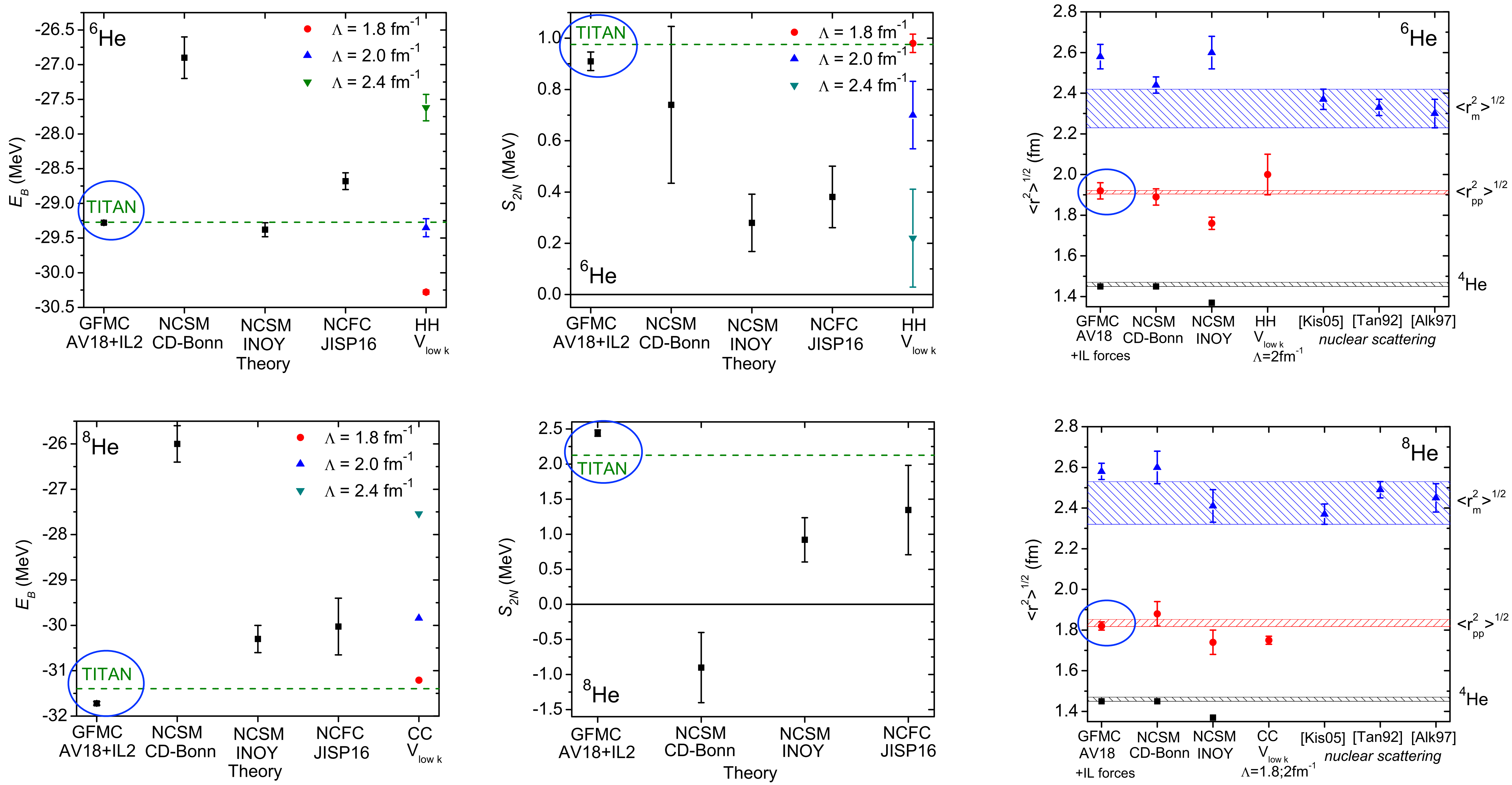
NCFC: No Core Full Configuration method (=NCSM) uses V2N only (JISP16)

HH: Hyperspherical Harmonic expansion, uses V2N only ($V_{\text{low k}}$)

CC: Coupled cluster theory, uses V2N only ($V_{\text{low k}}$)

Comparison with GFMC

GFMC, AV18 and ILs:

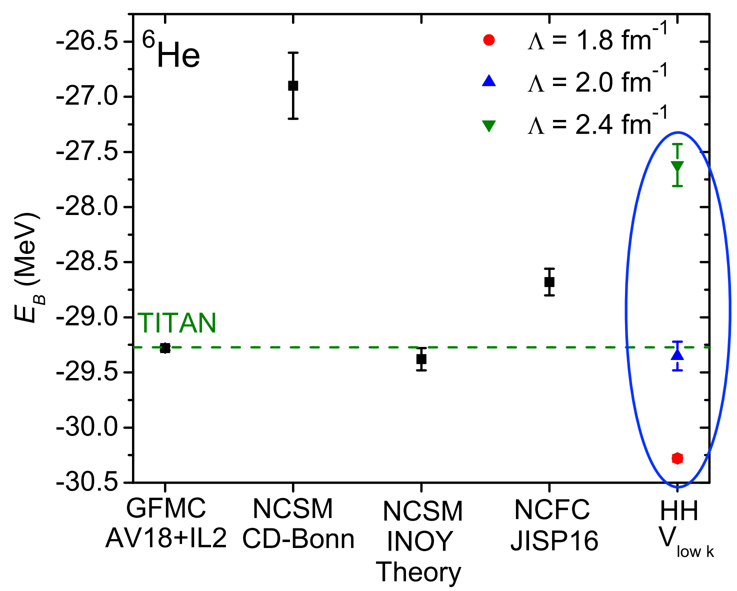


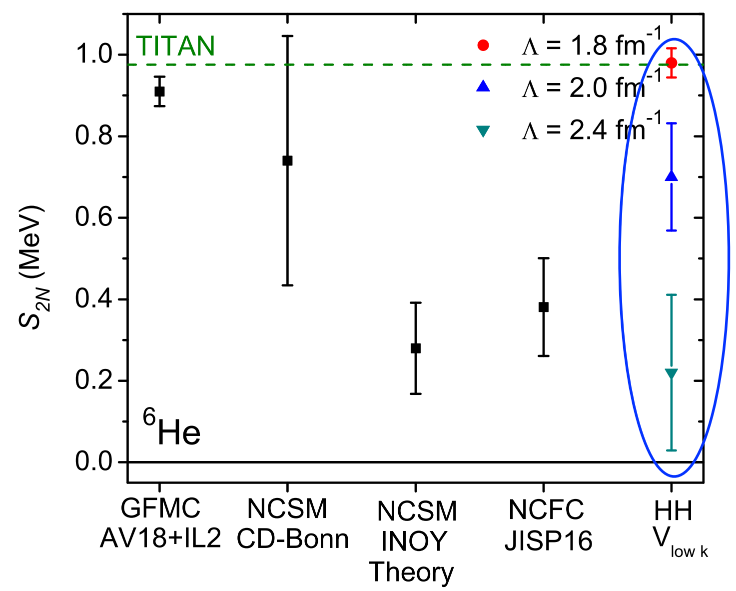
→ Method that provides the closest values to experiment

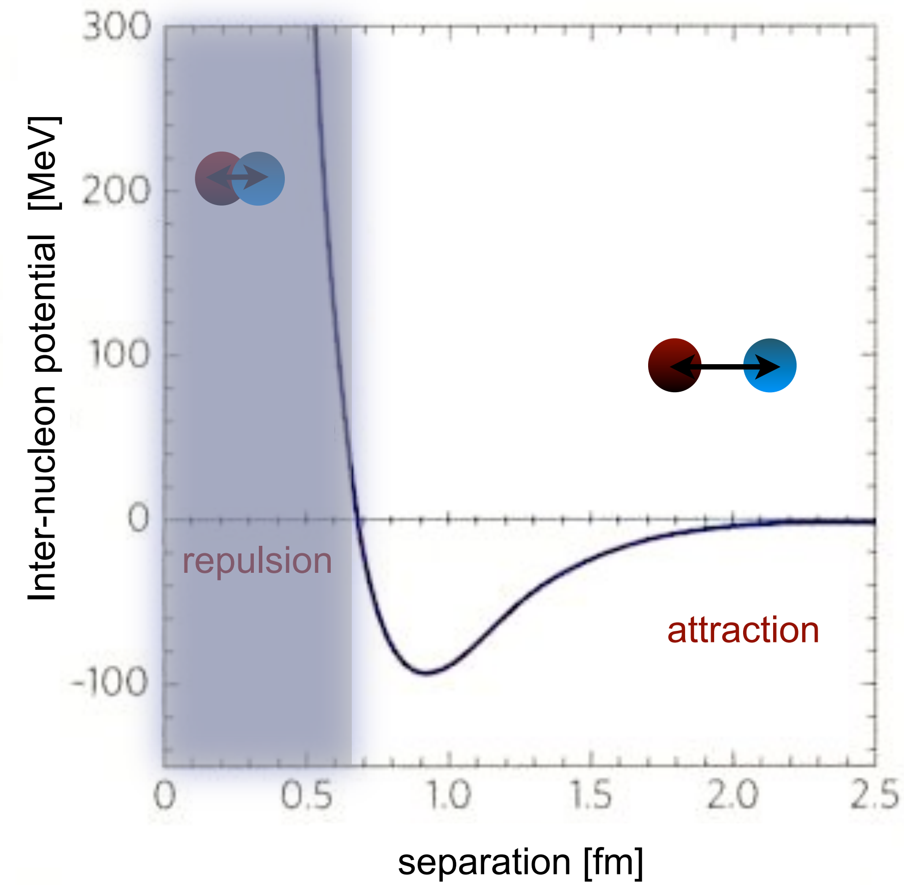
→ Only method that uses V3N

Comparison with HH & CC

HH and CC, $V_{low\ k}$:

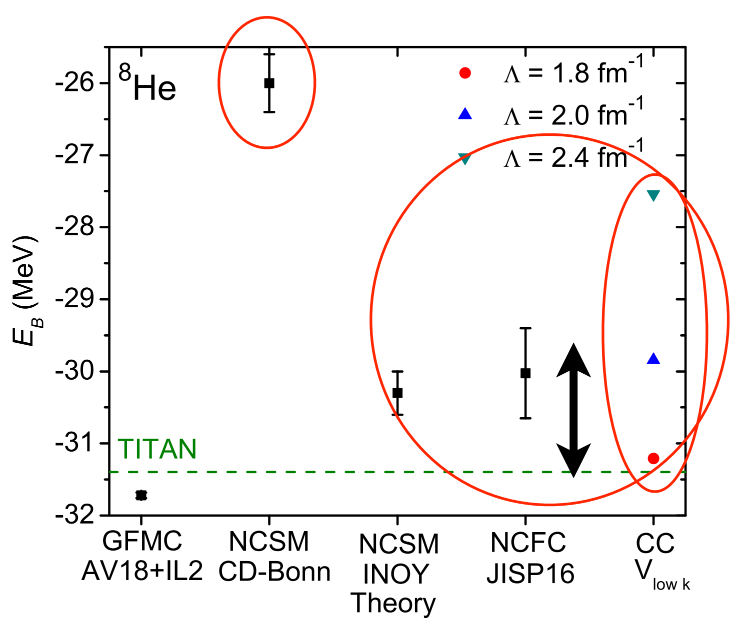


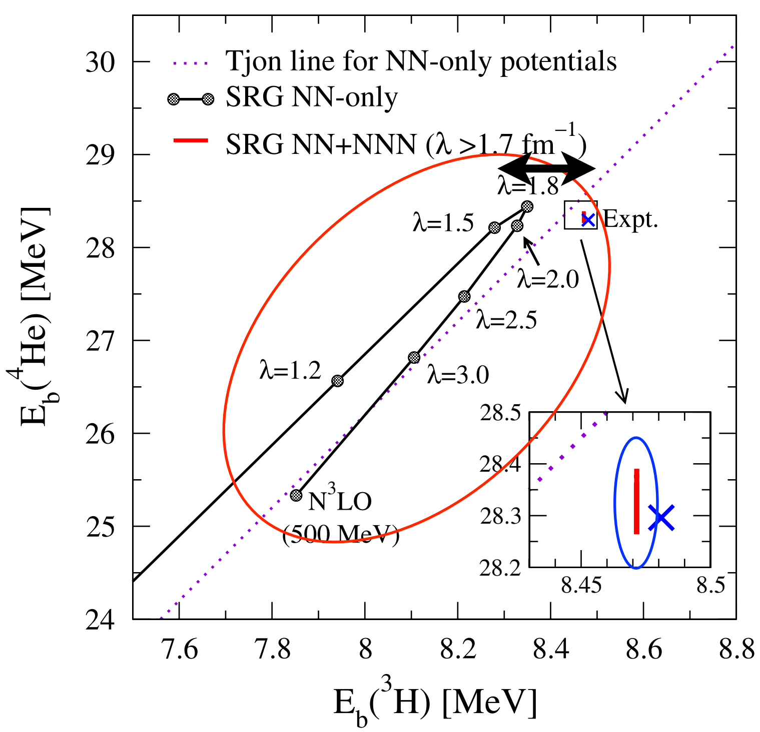




$$\Lambda \sim \frac{1}{r}$$

$$H(\Lambda) = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \ldots$$





- Only methods to vary the cut-off Λ for ^{6,8}He
- Allows us to estimate the effect of missing V3N
- Minimal change in ^{4}He E_B when V3N are included

- Cannot accurately predict the E_B when only V2N are used