# *Fermi matrix elements and nucleon-nucleus scattering*


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- motivation
- Coulomb corrections ($\delta_C$) to SBD using SM
- $\delta_C$ and scattering using a Green's function approach

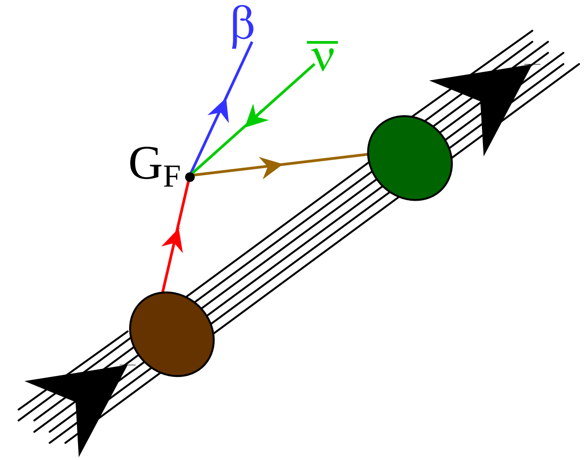

***Collaborators:***

***W. H. Dickhoff,***

***B. K. Jennings***

## Superallowed β decay:

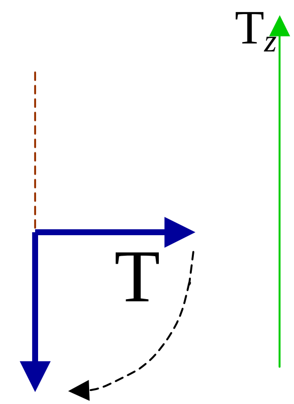
$$ft = \frac{K}{G_V^2 \, |M_F|^2}$$




- ($J^\pi$=0+, T=1, $T_z$) → ($J^\pi$=0+, T=1, $T_z$±1)
- rotation in isospin space!

$$M_{F0} = \langle T, T_z \pm 1 | T_\pm | T, T_z \rangle$$

$$= \sqrt{T(T+1) - T_z(T_z \pm 1)} = \sqrt{2}$$




## Corrections:

- isospin breaking:

$$|M_F|^2 = 2\ (1 - \delta_C)$$

- radiadive

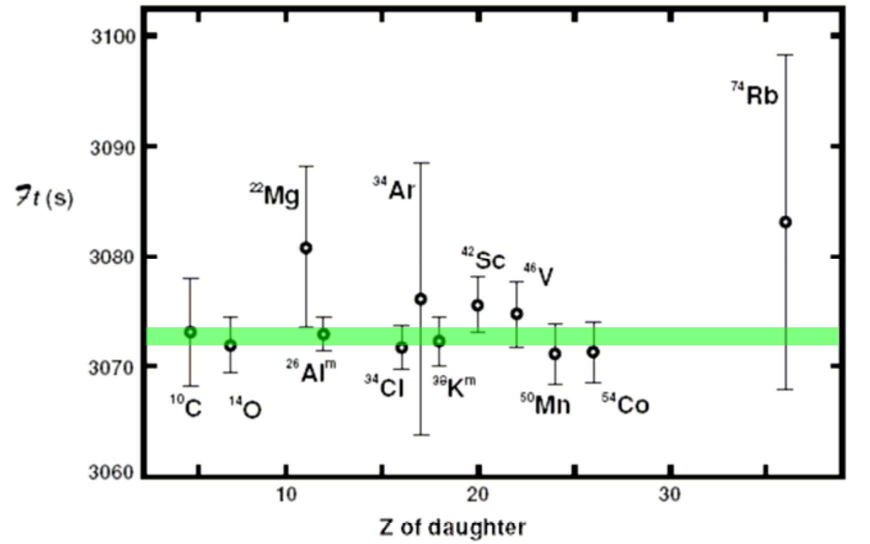
# Conserved vector current (CVC) and CKM unitarity

- CVC hypothesis:

$$\mathcal{F}t = ft\,(1+\delta_R')\,(1+\delta_{NS}-\delta_C) = \frac{K}{2\,G_V^2\,(1+\Delta_R^V)} = const$$

measurements

nucleus dependent effects




- CKM unitarity

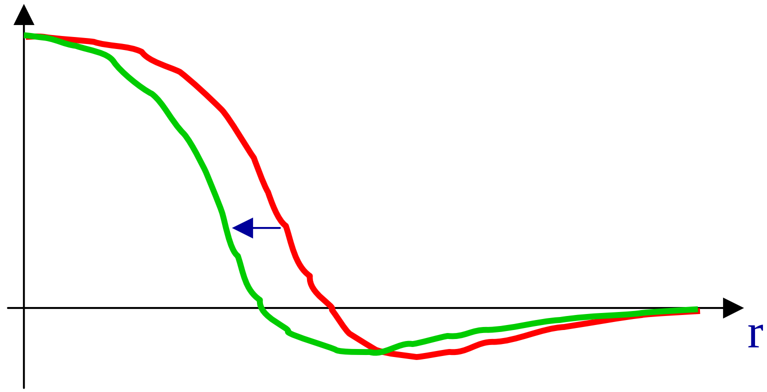
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9966(14) \quad 2.4\sigma \text{ deviation}$$

…but see new results for $V_{us}$…

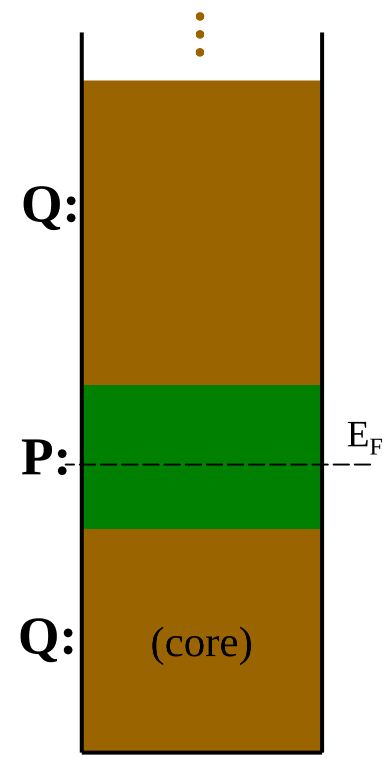
[PRC **71**, 055501 (2005)]

# Coulomb corrections ($\delta_C$)

- Change in orbital size due to Coulomb repulsion




- Split the Hilbert space in two:
  - P: SM calculation with effective interaction → $\delta_{IM}$
  - Q: radial overlaps → $\delta_{RO}$




- Total correction: $\delta_C = \delta_{IM} + \delta_{RO}$

# Coulomb corrections – available results

- Hardy & Towner [PRC **66**, 035501 (2002)]
  - Shell model → fitting IMME
  - Woods Saxon radial shapes
- Ormand & Brown [PRC **52**, 2455 (1995)]
  - Shell Model → INC fitted to data
  - Skyrme HF radial functions

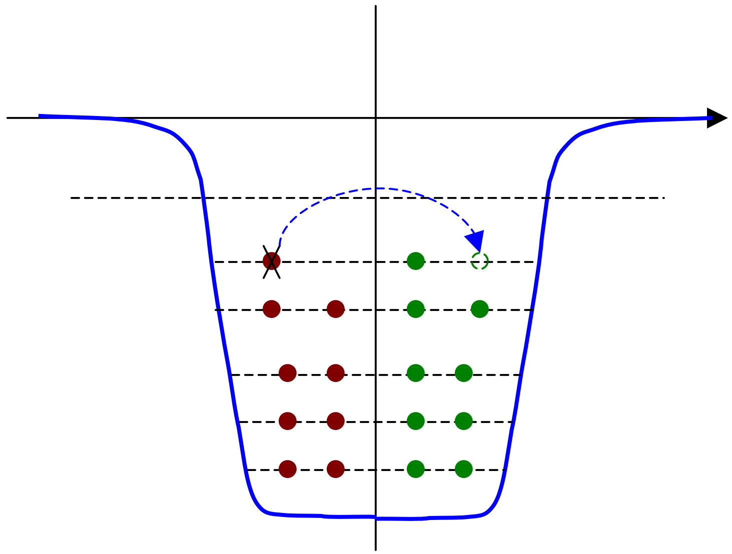
…fits to isospin properties

→Both are in agreement with CVC, but in disagreement with each other

- Other possible effects:
  - Orbital depletion, spectral distribution
  - Particle states in the continuum
  - Opening of the shell core

→ a third, independent, calculation would be helpful

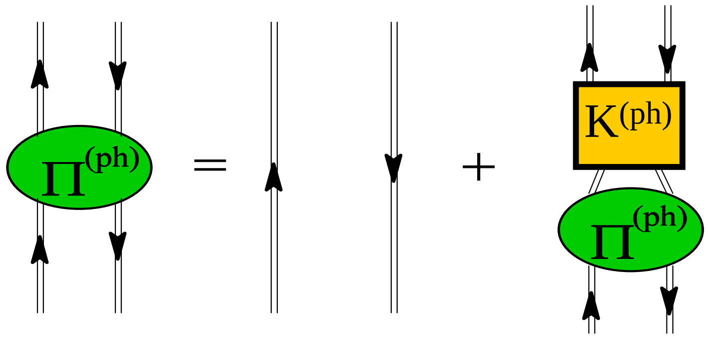
# Computing $\delta_C$ using RPA



- Model as a particle-hole excitation
- Isospin breaking (INC) interaction:
  - It is weak → (D)RPA
  - Can use modern NN potentials
  - Other constraints from isospin data
- Nuclear structure effects:
  - Probability of adding/removing a neutron/proton from the orbit
    → scattering.
  - Radial shape
- …planned experiments on $^{26m}$Al at TRIUMF

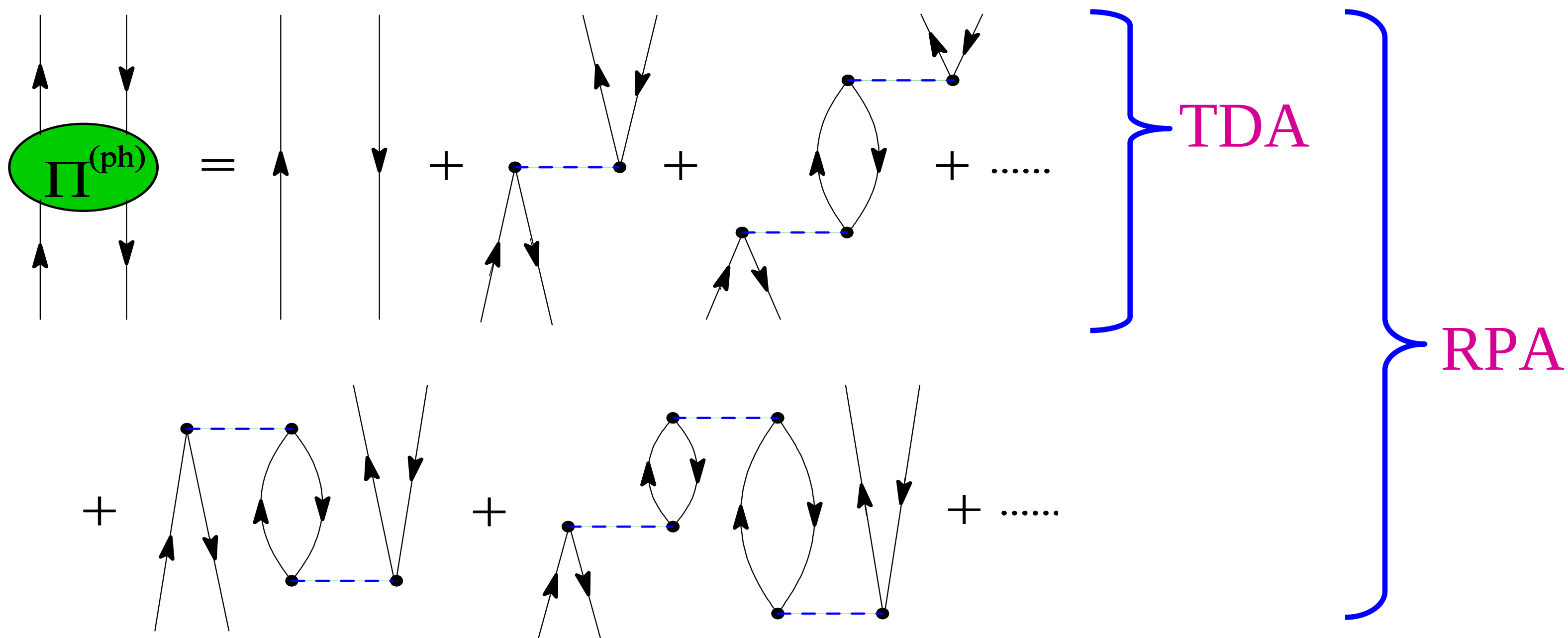
# Collective excitations

- ph propagator:



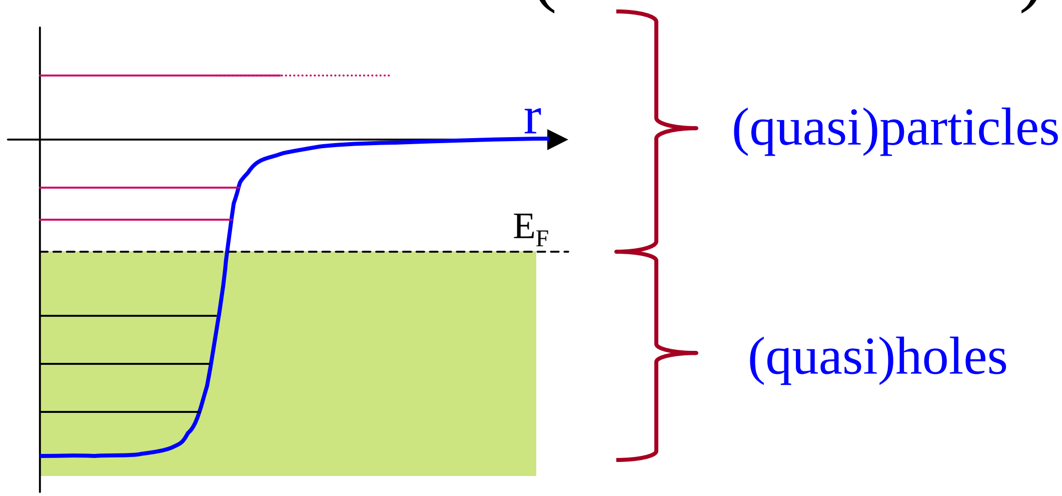

(Bethe-Salpeter Equation)

- RPA approximation:

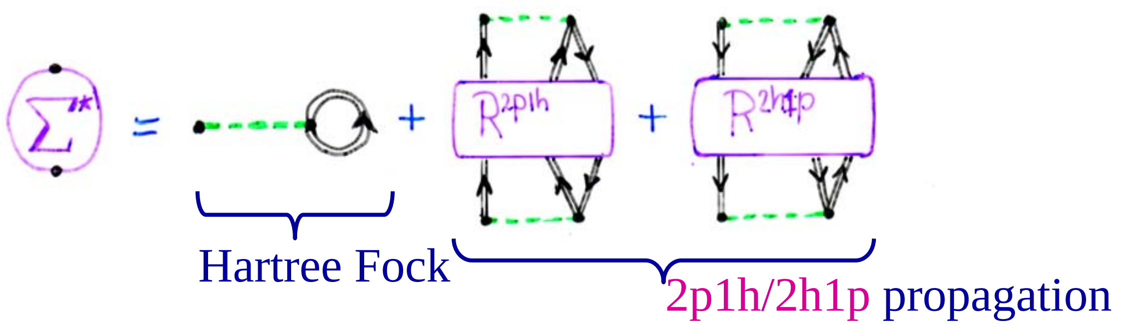


# Optical model: p vs h states

- Particle states versus (Pauli forbidden) holes



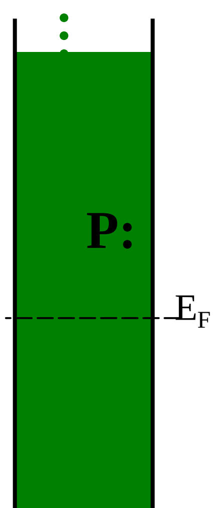

- Optical model in p-h space (nuclear self-energy)

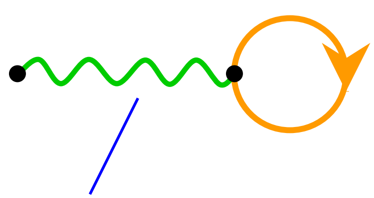



[see e.g. Mahaux and Sartor, Adv. Nucl. Phys. 20, (1991), Jennings, Escher PRC (2002)]

# *Approximation to the nuclear self-en. - I*

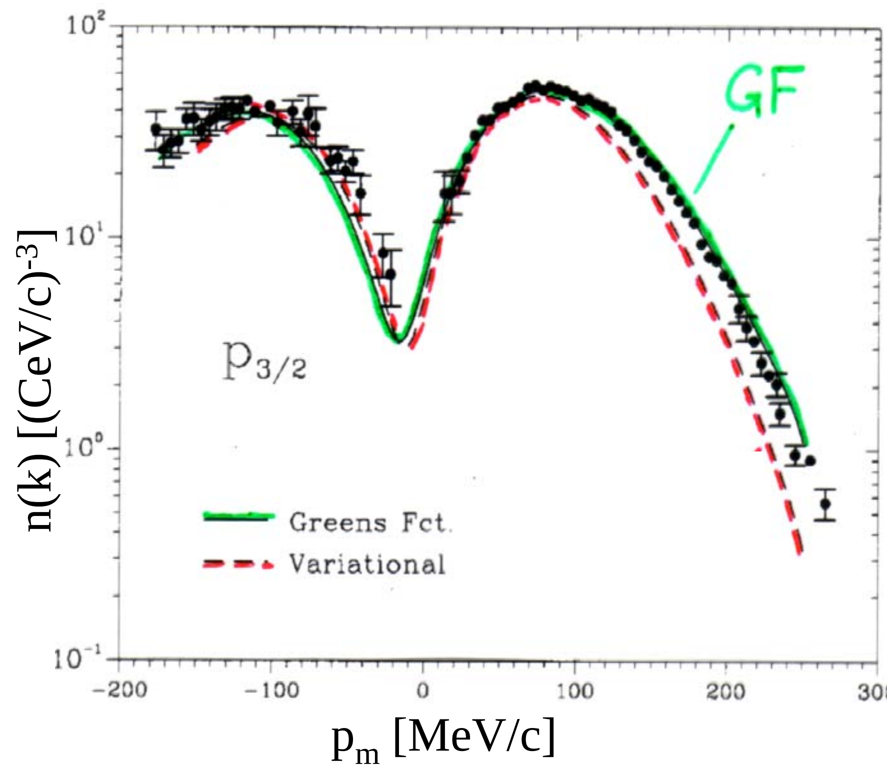
- Full k- Hilbert space
- Mean-field only
- Use G-matrix (→short-range physics only is included)



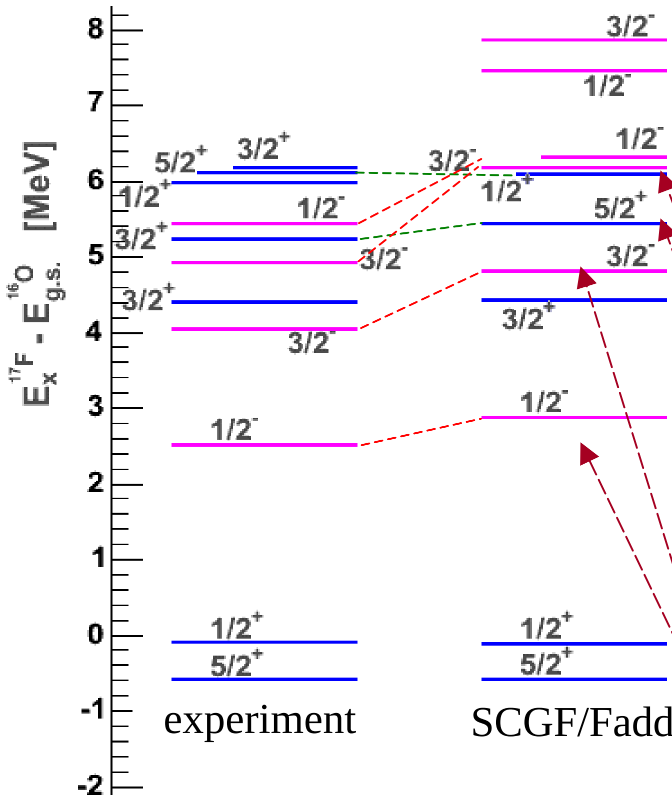
in-medium
G-matrix

$p_{3/2}$ "quasihole" *wave function*
compared to $^{16}O(e,e'p)$ data

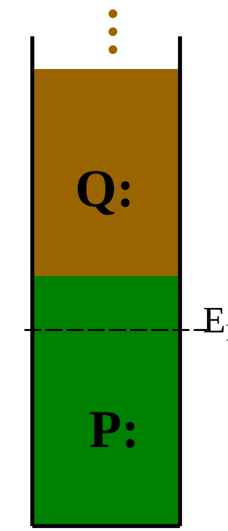



[Radici, et al. PRC55, 810 (1997)]

# *Approximation to the nuclear self-en. - II*

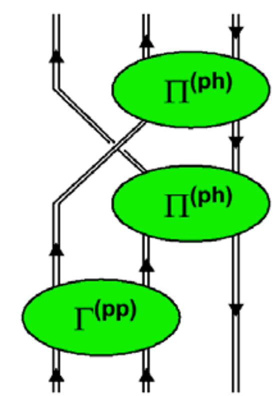



- Truncated model space (no core)
- Coupling particles to collective excitations (→ long-range physics)



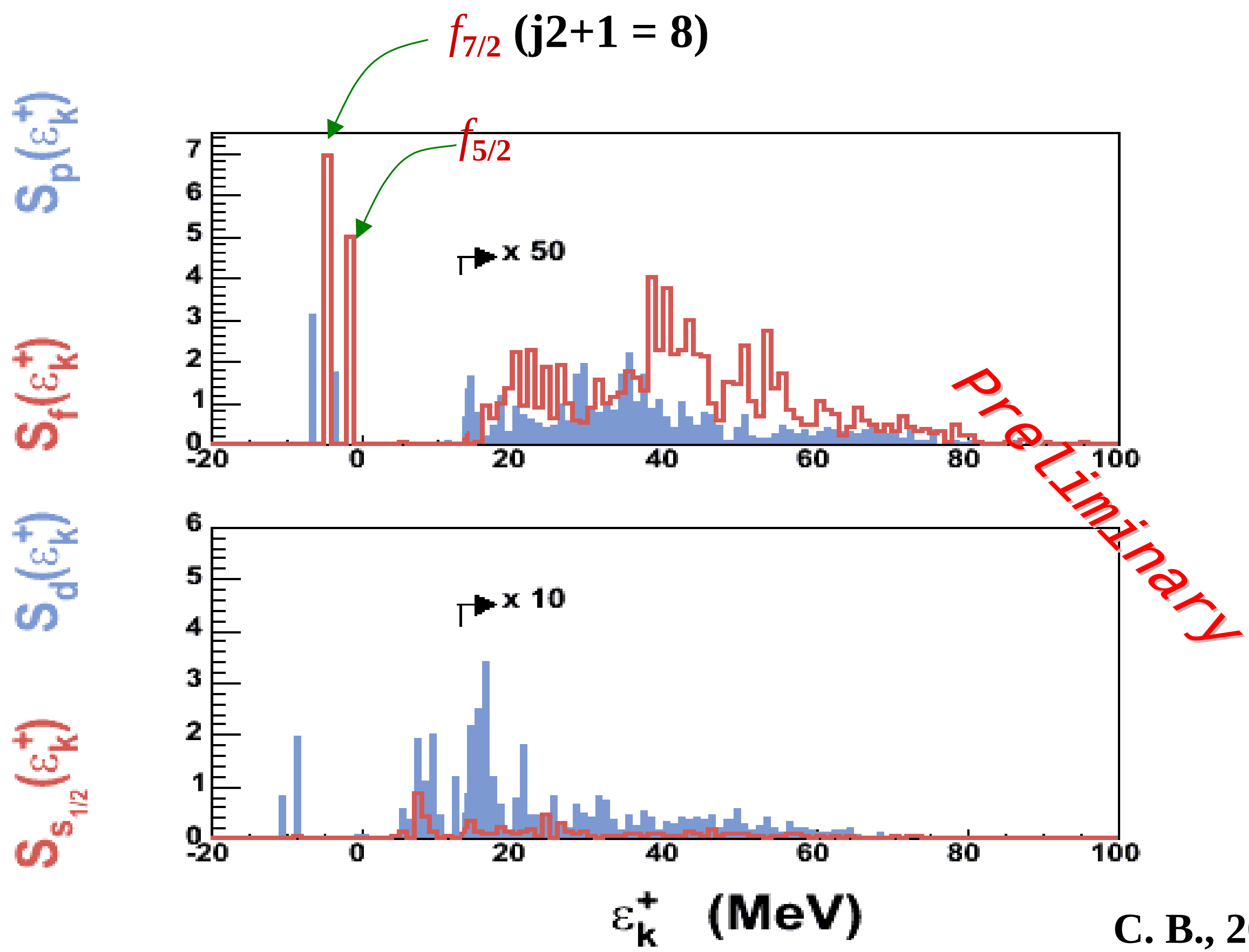

**particle on the first 0+ excited state**

**coupling to 3- and 1- phonons in $^{16}O$**




[CB, et al., PRC63, 034313 (2001)
PRC65, 064313 (2002)]

# Example of $^{32}S$ spectral distribution




C. B., 2005

# Mixing the two: model for the opt. potential

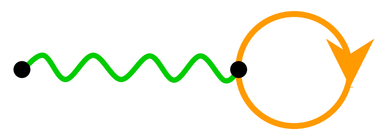
• From Faddeev/SCGF calculations:

$$\Sigma^*_P(r,r';\omega) = \sum_{\alpha,\beta\in P} \phi_\alpha(r) \left[ \Sigma^{MF}_{\alpha\beta}(\omega) + \sum_{n+} \frac{m^{n+}_\alpha \, m^{n+}_\beta}{\omega-\varepsilon^{n+}+i\eta} + \sum_{k-} \frac{m^{k-}_\alpha \, m^{k-}_\beta}{\omega-\varepsilon^{k-}-i\eta} \right] \phi^*_\beta(r')$$

→the (small) model space P is inadequate.

• take the missing components from the mean-field (as discussed before):

$$\Sigma^*_{MF}(r,r';\omega) =$$



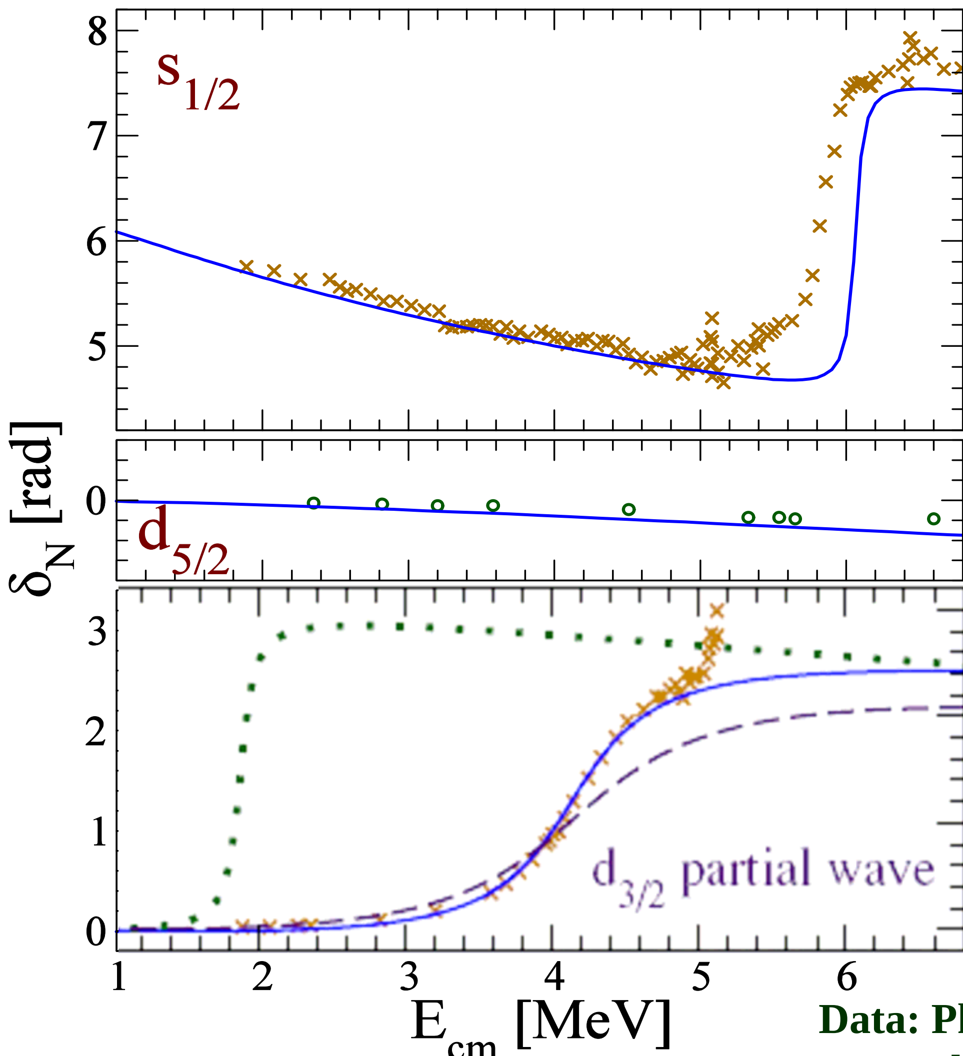
Dickhoff, Müther, Polls, PRC51 3040 (95)

•Then

$$\Sigma^*(r,r';\omega) = \Sigma^*_{MF}(r,r';\omega) + \Sigma^*_P(r,r';\omega)$$

•…let's see how well this can describe the nucleon scattering

# p-$^{16}$O phase shifts – positive parity waves




**(C.B., B.Jennings, accepted by Phys. Rev. C)**

- The depth of the mean field potential is modified to reproduce s.p. energies
- The phase shift are in agreement with the experiment!
- Non-MF resonances
- p waves are not as good
→ *more work required*.

**Data: Phys. Rev. 126, 2147 ('62)**
**Phys. Rev. 137, 284B ('64)**

# Summary

- The same formalism describes scattering phase shifts and Fermi matrix elements
  - May use scattering data to constrain the model and predict coulomb corrections for superallowed β decay..

- Phenomenological correction will STILL be needed. However:
  - Fit to a different set on data
  - Possibly, more theory in the INC part of the interaction
  - Independent evaluation (and complimentary to SM approach)

- Extensions of codes and formalism under development.

- NOTE: Self-consistent Green's function (SCGF) will be applied to a larger set of nuclear structure problems, in particular toward the drip lines (two-body emission, particle transfer, excited spectra, spectroscopic factors, spectral distributions, etc…)

***Review of SCGF for nuclei: W. H. Dickhoff, C.B., Prog. Part. Nucl. Phys. 52, 377 (2004).***

# One-body Green's function

- Nuclear many-body problem:

$$\hat{H}=\sum_{\alpha,\beta}\langle\alpha|\frac{\hat{p}^2}{2m}|\beta\rangle c_\beta^+ c_\alpha + \frac{1}{4}\sum_{\alpha,\beta}\langle\alpha\beta|\hat{V}^{2N}|\gamma\delta\rangle c_\alpha^+ c_\beta^+ c_\delta c_\gamma + \ldots$$

$$\hat{H}|\Psi_n^A\rangle = E_n^A|\Psi_n^A\rangle$$ n[th] excited state with A nucleons

- One-body Green's function:

$$g_{\alpha b}(t-t') = -i\ \langle\Psi_n^A|\mathrm{T}[c_\alpha(t)\ c_\beta^+(t')]|\Psi_o^A\rangle$$

- t > t' → propagation of a quasiparticle
- t < t' → propagation of a quasihole

# and in Lehmann (energy) representation:

- $$g_{\alpha\beta}(\omega) = \frac{1}{2\pi}\int d\tau\, e^{-i\omega\tau}\; g_{\alpha\beta}(\tau)$$

$$= \sum_n \frac{\langle\Psi_0^A | c_\alpha | \Psi_n^{A+1}\rangle\langle\Psi_n^{A+1} | c_\beta^+ | \Psi_0^A\rangle}{\omega - (E_n^{A+1} - E_0^A) + i\eta} + \sum_k \frac{\langle\Psi_0^A | c_\beta^+ | \Psi_k^{A-1}\rangle\langle\Psi_k^{A-1} | c_\alpha | \Psi_0^A\rangle}{\omega - (E_0^A - E_k^{A-1}) - i\eta}$$

  - forward-going part (A+1 nucleons): quasiparticles
  - backward-going part (A-1nucleons): quasiholes

- Spectral function

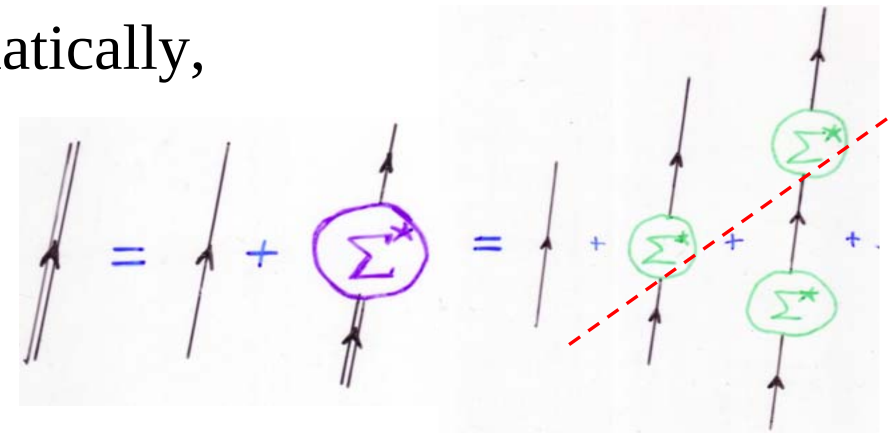
$$S_\alpha(\omega) = \frac{1}{\pi}\,\mathrm{Im}\, g_{\alpha\alpha}(\omega)$$

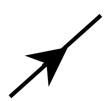
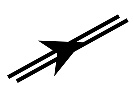
- Equation of motion (in Dyson-Schwinger form…)

# Dyson Equation

- $g_{\alpha\beta}(\omega) = g^{o}_{\alpha\beta}(\omega) + \sum_{\gamma,\delta} g^{o}_{\alpha\gamma}(\omega)\ \Sigma^{*}_{\gamma\delta}\ g_{\delta\beta}(\omega)$

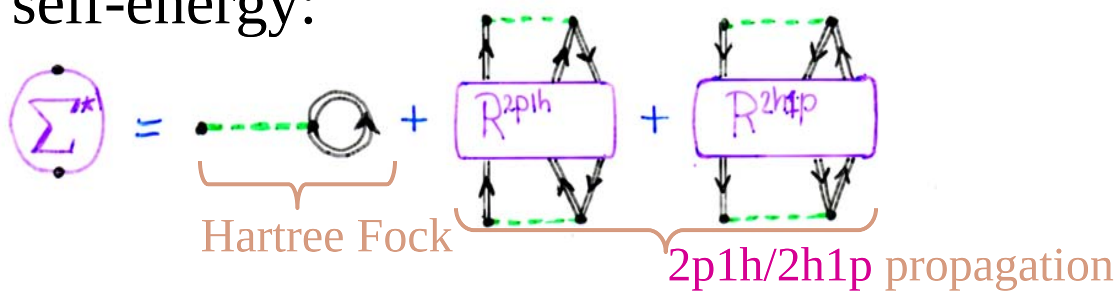
- diagrammatically,



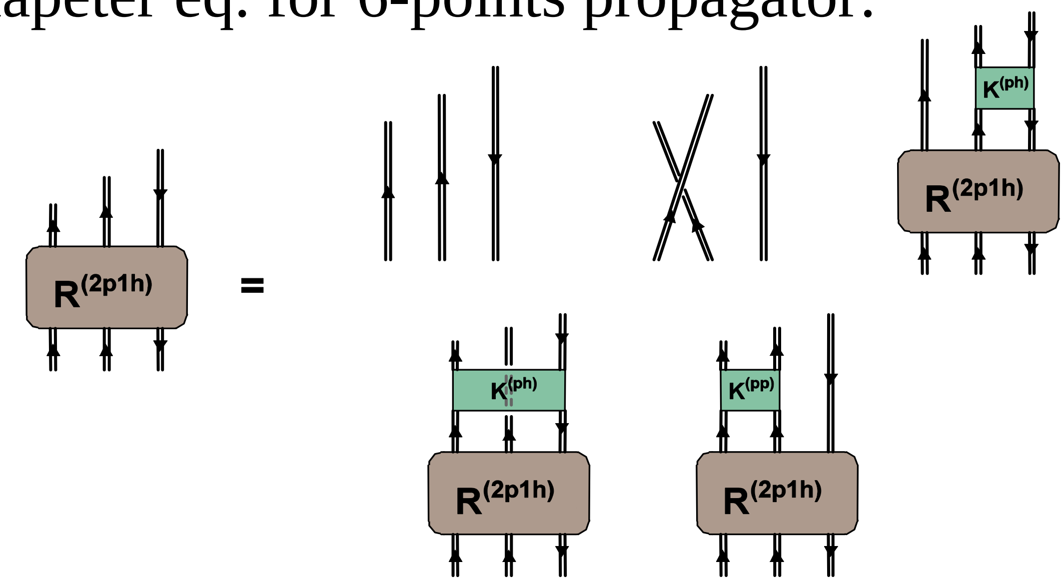
- $g^0(\omega)$ =  is the unperturbed one-body propagator
- $g(\omega)$ =  is the full one-body propagator
- There exist a hierarchy of relations between higher order Green's functions

# Non perturbative expansion of the self-energy

- Irreducible self-energy:




- Bethe-Slapeter eq. for 6-points propagator:

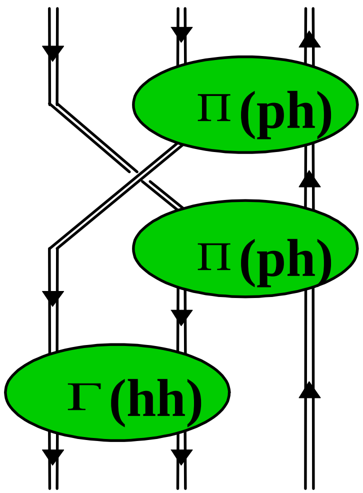



- *non-perturbative* expansion of the BSE kernel in terms of higher order Green's functions… where to truncate?

# Coupling of single-particle to collective ph and 2p(2h) phonons

**[Barbieri, et al., PRC63, 034313 (2001)]**

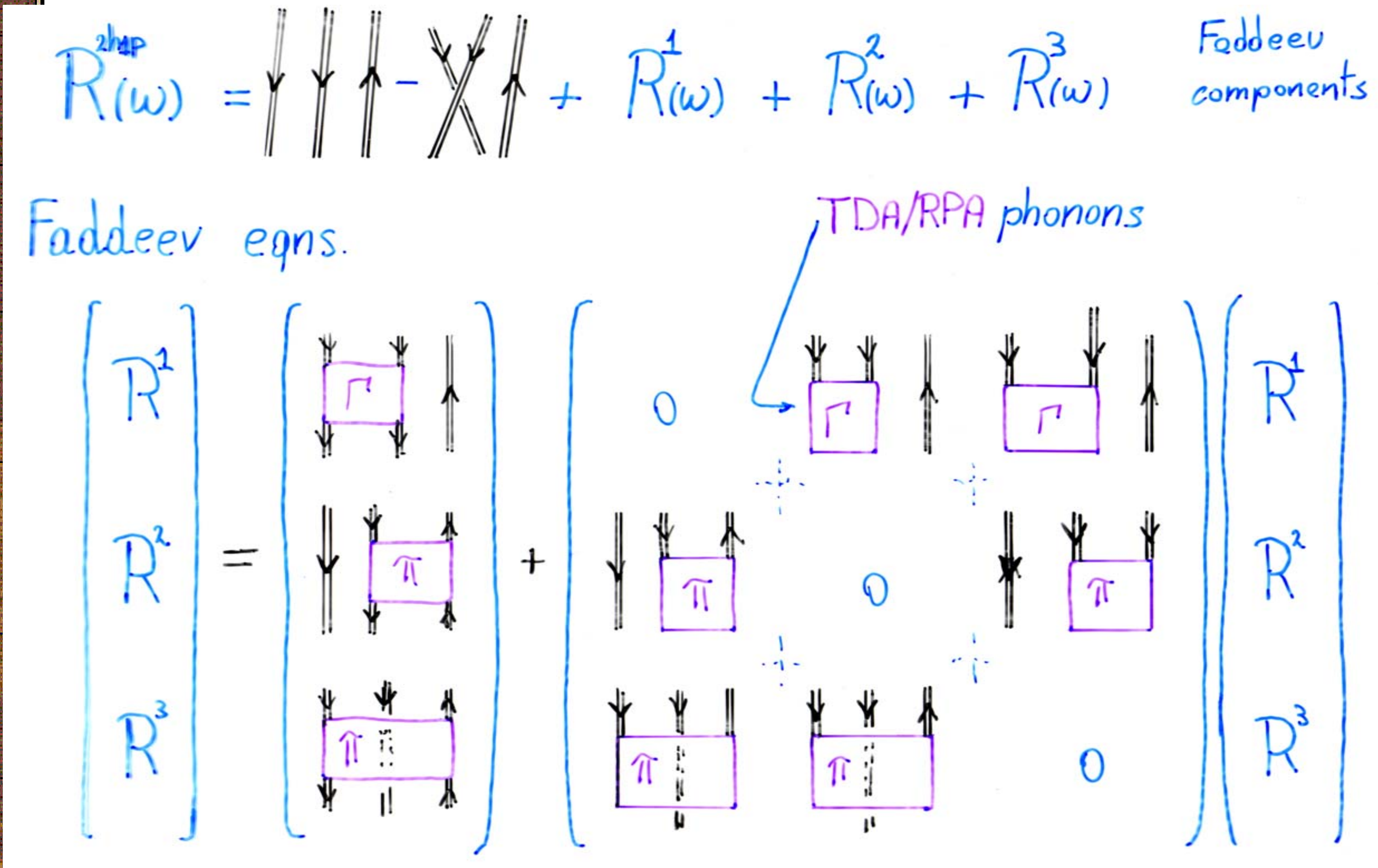
- Truncation of the self-energy at a level that includes between couplings of single particle and collective phonons
- This lead naturally to a set of Faddeev equations:




- *all order* summation
- Pauli contributions (up to 2p1h/h1p)
- Phonon in RPA approx. (and beyond: BSE)

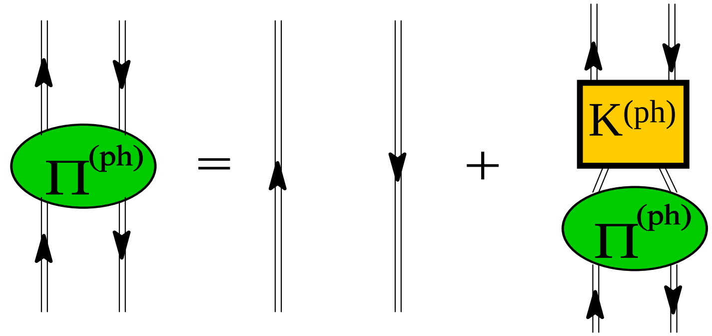
- Expansion in terms of g(ω)=  → conservation of basic sum rules (see Baym and Kadanoff, ‘50s)

# Faddeev equations for the 2h1p motion



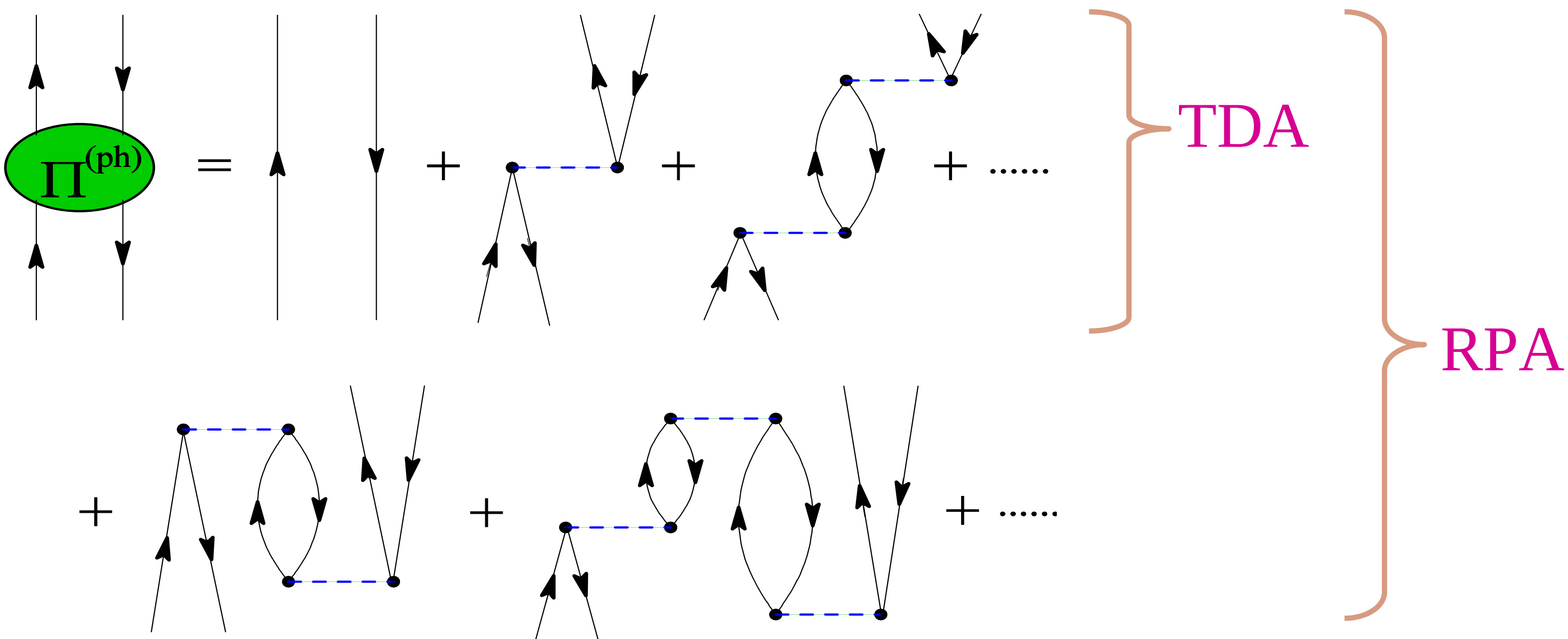
# Collective phonons

- ph propagator:




(Bethe-Salpeter Equation)

- RPA approximation:



# Self-consistent Green's function scheme (SCGF)